Introduction

PRINCIPLES AND TOOLS OF STATISTICS

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MPIFR, BONN, ROOM 0.02

BAYESIAN APPROACH
(MUCH MORE IN PRACTICAL EXAMPLE)

Likelihood

Posterior

Prior

$$P(\theta \mid \text{data}) = \frac{\mathrm{P}(\text{data} \mid \theta) \times \mathrm{P}(\theta)}{\mathrm{P}(\text{data})}$$

Evidence

Prior: Our information about the probability distribution of model parameters before we did the experiment
Posterior: the updated probability distribution of model parameters
Likelihood: Our model and its perspective on the new data
Evidence: The probability density of the new data integrated over all possible parameters of the model - a normalisation factor

BAYESIAN APPROACH
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Likelihood

Posterior

Prior

$$P(\theta\,|\,\text{data}) = \frac{\mathrm{P}(\text{data}\,|\,\theta) \times \mathrm{P}(\theta)}{\mathrm{P}(\text{data})}$$

Evidence

Metropolis algorithm: The transition probabilities of the Markov chain are chosen according to

$$r = \begin{cases} 1, & \text{if } \ \mathrm{p}(\theta_{\mathrm{t+1}}\,|\,\text{data}) \geq \mathrm{p}(\theta_{\mathrm{t}}\,|\,\text{data}) \\ \dfrac{p(\theta_{t+1}\,|\,data)}{p(\theta_t\,|\,data)} & \text{if } \ \mathrm{p}(\theta_{\mathrm{t+1}}\,|\,\text{data}) < \mathrm{p}(\theta_{\mathrm{t}}\,|\,\text{data}) \end{cases}$$

SUMMARY

Things we have discussed:

- Foundation of random spaces and statistical models
- Random variables, distributions, and sampling from distributions
- Conditional probabilities, Bayes' rule and independence
- Probabilities on product spaces
- Expectation values
- Probabilistic convergence, laws of large numbers, central limit theorem
- Markov chains and sampling from distributions
- Likelihoods
- Confidence intervals
- Bayesian statistics, MCMC, credible intervals

SUMMARY

Things we have done:

- Set up a Conda environment, installed Astroconda, used Jupyter notebooks
- Used git version control
- Used various packages including numba, uncertainties, pandas, numpy.random, scipy.stats, scipy.optimize, emcee...
- Learned how to sample from distributions, with standard implementations and inverse CDF method as well as rejection sampling and MCMC
- Monte-Carlo-simulated error propagation for pathological cases
- Made empirical distribution functions from data and implemented sampling schemes from empirical distributions
- Studied a simple Markov chain
- Learned how to do regression and chi^2-minimization
- Learned how to find the maximum likelihood value
- Used emcee to fit a more complex model of the stellar orbit of S-2 including multi-threading