
Introduction

PRINCIPLES AND TOOLS OF STATISTICS

BAYESIAN APPROACH (MUCH MORE IN PRACTICAL EXAMPLE)

The diagram shows the Bayes' theorem equation with handwritten labels and arrows. The equation is $P(\theta | \text{data}) = \frac{P(\text{data} | \theta) \times P(\theta)}{P(\text{data})}$. The label 'Posterior' has an arrow pointing to the left side of the equation. The label 'Likelihood' has an arrow pointing to the numerator term $P(\text{data} | \theta)$. The label 'Prior' has an arrow pointing to the numerator term $P(\theta)$. The label 'Evidence' has an arrow pointing to the denominator term $P(\text{data})$.

$$P(\theta | \text{data}) = \frac{P(\text{data} | \theta) \times P(\theta)}{P(\text{data})}$$

Prior: Our information about the probability distribution of model parameters before we did the experiment

Posterior: the updated probability distribution of model parameters

Likelihood: Our model and its perspective on the new data

Evidence: The probability density of the new data integrated over all possible parameters of the model - a normalisation factor

BAYESIAN APPROACH (MUCH MORE IN PRACTICAL EXAMPLE)

The diagram illustrates the Bayesian formula with handwritten labels in red. The formula is $P(\theta | \text{data}) = \frac{P(\text{data} | \theta) \times P(\theta)}{P(\text{data})}$. Red arrows point from the labels to the corresponding parts of the formula: 'Likelihood' points to $P(\text{data} | \theta)$, 'Prior' points to $P(\theta)$, 'Evidence' points to $P(\text{data})$, and 'Posterior' points to $P(\theta | \text{data})$.

$$P(\theta | \text{data}) = \frac{P(\text{data} | \theta) \times P(\theta)}{P(\text{data})}$$

Metropolis algorithm: The transition probabilities of the Markov chain are chosen according to

$$r = \begin{cases} 1, & \text{if } p(\theta_{t+1} | \text{data}) \geq p(\theta_t | \text{data}) \\ \frac{p(\theta_t | \text{data})}{p(\theta_{t+1} | \text{data})} & \text{if } p(\theta_{t+1} | \text{data}) < p(\theta_t | \text{data}) \end{cases}$$



SUMMARY

Things we have discussed:

- Foundation of random spaces and statistical models
 - Random variables, distributions, and sampling from distributions
 - Conditional probabilities, Bayes' rule and independence
 - Probabilities on product spaces
 - Expectation values
 - Probabilistic convergence, laws of large numbers, central limit theorem
 - Markov chains and sampling from distributions
 - Likelihoods
 - Confidence intervals
 - Bayesian statistics, MCMC, credible intervals
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SUMMARY

Things we have done:

- Set up a Conda environment, installed Astroconda, used Jupyter notebooks
 - Used git version control
 - Used various packages including numba, uncertainties, pandas, numpy.random, scipy.stats, scipy.optimize, emcee...
 - Learned how to sample from distributions, with standard implementations and inverse CDF method as well as rejection sampling and MCMC
 - Monte-Carlo-simulated error propagation for pathological cases
 - Made empirical distribution functions from data and implemented sampling schemes from empirical distributions
 - Studied a simple Markov chain
 - Learned how to do regression and χ^2 -minimization
 - Learned how to find the maximum likelihood value
 - Used emcee to fit a more complex model of the stellar orbit of S-2 including multi-threading
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