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Trigonometric Parallaxes of Massive Star Forming Regions: VI. Galactic Structure, Fundamental Parameters and Non-Circular Motions

M. J. Reid¹, K. M. Menten², X. W. Zheng³, A. Brunthaler², L. Moscadelli⁴, Y. Xu^{2,5}, B. Zhang³, M. Sato^{1,6}, M. Honma⁶, T. Hirota⁶, K. Hachisuka⁷, Y. K. Choi², G. A. Moellenbrock⁸, & A. Bartkiewicz⁹

ABSTRACT

We are using the NRAO Very Long Baseline Array (VLBA) and the Japanese VERA project to measure trigonometric parallaxes and proper motions of masers found in high-mass star-forming regions across the Milky Way. Early results from 18 sources locate several spiral arms. The Perseus spiral arm has a pitch angle of $16^\circ \pm 3^\circ$, which favors four rather than two spiral arms for the Galaxy. Combining distances, proper motions, and radial velocities yields complete 3-dimensional kinematic information. We find that star forming regions on average are orbiting the Galaxy $\approx 15 \text{ km s}^{-1}$ slower than expected for circular orbits. By fitting the measurements to a model of the Galaxy, we estimate the distance to the Galactic center $R_0 = 8.4 \pm 0.6 \text{ kpc}$ and a circular rotation speed $\Theta_0 = 254 \pm 16 \text{ km s}^{-1}$. The ratio Θ_0/R_0 can be determined to higher accuracy than either parameter individually, and we find it to be $30.3 \pm 0.9 \text{ km s}^{-1} \text{ kpc}^{-1}$, in good agreement with

¹Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

²Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, 53121 Bonn, Germany

³Department of Astronomy, Nanjing University Nanjing 210093, China

⁴Arcetri Observatory, Firenze, Italy

⁵Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210008, China

⁶VERA Project, National Astronomical Observatory, Tokyo 181 8588, Japan

⁷Shanghai Astronomical Observatory, 80 Nandan Rd., Shanghai, China

⁸National Radio Astronomy Observatory, Socorro, NM, USA

⁹Torun Centre for Astronomy, Nicolaus Copernicus University, Gagarina 11, 87-100 Torun, Poland

the angular rotation rate determined from the proper motion of Sgr A*. The data favor a rotation curve for the Galaxy that is nearly flat or slightly rising with Galactocentric distance. Kinematic distances are generally too large, sometimes by factors greater than two; they can be brought into better agreement with the trigonometric parallaxes by increasing Θ_0/R_0 from the IAU recommended value of $25.9 \text{ km s}^{-1} \text{ kpc}^{-1}$ to a value near $30 \text{ km s}^{-1} \text{ kpc}^{-1}$. We offer a “revised” prescription for calculating kinematic distances and their uncertainties, as well as a new approach for defining Galactic coordinates. Finally, our estimates of Θ_0 and of Θ_0/R_0 , when coupled with direct estimates of R_0 , provide evidence that the rotation curve of the Milky Way is similar to that of the Andromeda galaxy, suggesting that the dark matter halos of these two dominant Local Group galaxy are comparably massive.

Subject headings: Galaxy: fundamental parameters, structure, kinematics and dynamics, halo — stars: formation — astrometry

1. Introduction

The Milky Way is known to possess spiral structure. However, revealing the nature of this structure has proved elusive for decades. The Georgelin & Georgelin (1976) study of HII regions produced what has been generally considered the “standard model” for the spiral structure of the Galaxy. However, after decades of study there is little agreement on this structure. Indeed, we do not really know the number of spiral arms (Simonson 1976; Cohen et al. 1980; Bash 1981; Vallée 1995; Drimmel 2000; Russeil, D. 2003) or how tightly wound is their pattern. The primary reason for the difficulty is the lack of accurate distance measurements throughout the Galaxy. Photometric distances are prone to calibration problems, which become especially severe when looking through the copious dust to distant objects in the plane of the Galaxy. Thus, most attempts to map the Galaxy rely on radio frequency observations and kinematic distances, which involve matching source Doppler shifts with line-of-sight velocities expected from a model of Galactic rotation. However, because of distance ambiguities in the first and fourth quadrants (where most of the spiral arms are found) and the existence of sizeable non-circular motions, kinematic distances can be highly uncertain (Burton & Bania 1974; Liszt & Burton 1981; Gómez 2006).

We are measuring trigonometric parallaxes and proper motions of sources of maser emission associated with high-mass star forming regions (HMSFRs), using the National

Radio Astronomy Observatory’s ¹ Very Long Baseline Array (VLBA) and the Japanese VERA project. The great advantage of trigonometric parallaxes and proper motions is that one determines source distances directly and geometrically, with no assumptions about luminosity, extinction, metallicity, crowding, etc. Also from the same measurements, one determines proper motions, and if the time sampling is optimal there is little if any correlation between the parallax and proper motion estimates. Thus, the magnitude of the proper motion does not affect the parallax accuracy. Combining all of the observational data yields the full 3-dimensional locations and velocity vectors of the sources.

Results for 12 GHz methanol masers toward 10 HMSFRs, carried out with the VLBA (program BR100), are reported in the first five papers in this series (Reid et al. 2009; Moscadelli et al. 2009; Xu et al. 2009; Zhang et al. 2009; Brunthaler et al. 2009), hereafter Papers I through V, respectively. Eight other sources with H₂O or SiO masers have been measured with VERA (Honma et al. 2007; Hirota et al. 2007; Choi et al. 2008; Sato et al. 2008) and with methanol, H₂O or continuum emission with the VLBA (Hachisuka et al. 2006; Menten et al. 2007; Moellenbrock, Claussen & Goss 2007; Bartkiewicz et al. 2008; Hachisuka et al. 2009). In this paper, we collect the parallaxes and proper motions from these papers in order to study the spiral structure of the Galaxy. Combining distances, Doppler shifts, and proper motions, allows us not only to locate the HMSFRs that harbor the target maser sources in 3-dimension, but also to determine their full (3-dimensional) space motions. In §2 we map the locations of the HMSFRs and measure the pitch angles of some spiral arms. In §3 we use the full 3-dimensional spatial and kinematic information to examine the non-circular (peculiar) motions of these star forming regions. Next, we fit the data with a model of the Galaxy and estimate the distance from the Sun to the Galactic center (R_0) and the circular orbital speed at the Sun (Θ_0). The nature of the rotation curve and its effect on estimates of R_0 and Θ_0 is also discussed. In §4 we compare kinematic distances with those determined by trigonometric parallax and offer a new prescription to improve such distance estimates. In §5 we discuss limitations of the current definition of Galactic coordinates and propose a new system based on dynamical information. Finally, we discuss the broader implications of our results in §6.

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2. Galactic Spiral Structure

Table 1 summarizes the parallax and proper motions of 18 regions of high-mass star formation measured with VLBI techniques. The locations of these star forming regions in the Galaxy are shown in Fig. 1, superposed on an artist’s conception of the Milky Way. Distance errors are indicated with error bars (1σ), but for most sources the error bars are smaller than the dots.

2.1. Spiral Arms

The HMSFRs with parallaxes locate several spiral arms. The three sources closest to the Galactic center (G 23.0–0.4, G 23.4–0.2, and G 23.6–0.1) appear to be members of the Crux-Scutum arm and possibly the Norma or the 3-kpc arm. However, the parallax uncertainties for these distant, low-declination sources are currently not adequate to clearly distinguish among these arms, especially in the crowded region where the Galactic bar (see Blitz & Spergel (1991b) and references therein) ends and the arms begin (Benjamin et al. 2005; Dame & Thaddeus 2008).

Three sources (G 35.2–0.7, G 35.2–1.7 & W 51 IRS 2) are in the Carina-Sagittarius arm, whose distance from the Sun is 2.5 kpc at $\ell \approx 35^\circ$.

Five sources (S 252, W3(OH), IRAS 00420+5530, NGC 281, and NGC 7538) clearly trace a portion of the Perseus arm, which is located between distances of 2.10 kpc at $\ell = 189^\circ$ (S 252) and 2.64 kpc at $\ell = 112^\circ$ (NGC 7538). NGC 281 is slightly offset from the other sources in the Perseus arm and is believed to be associated with a super-bubble (Sato et al. 2008). As such, it may not accurately trace spiral structure.

Two sources, (S 269 & WB 89-437), measured by Honma et al. (2007) with the VERA array and Hachisuka et al. (2009) with the VLBA, lie beyond the Perseus arm and begin to trace an Outer (Cygnus) arm at a distance from the Sun of 5.3 kpc at $\ell = 196^\circ$ for S 269 to 5.9 kpc at $\ell = 135^\circ$ for WB 89-437.

The remaining five sources (G 232.6+1.0, VY CMa, Orion, Cep A and G59.7+0.1) trace the Local (Orion) “arm,” which appears to be a spur between the Carina–Sagittarius and Perseus arms. The Sun is in or near this spur, and we can trace it between G 59.7+0.1 near the Carina–Sagittarius arm at $\ell = 60^\circ$ and G 232.6+1.0 near the Perseus arm at $\ell = 233^\circ$.

Table 1. Parallaxes & Proper Motions of High-mass Star Forming Regions

Source	l (deg)	b (deg)	Parallax (mas)	μ_x (mas y ⁻¹)	μ_y (mas y ⁻¹)	v_{LSR} (km s ⁻¹)	Ref.
G 23.0–0.4...	23.01	–0.41	0.218 ± 0.017	-1.72 ± 0.04	-4.12 ± 0.30	$+81 \pm 3$	V
G 23.4–0.2...	23.44	–0.18	0.170 ± 0.032	-1.93 ± 0.10	-4.11 ± 0.07	$+97 \pm 3$	V
G 23.6–0.1...	23.66	–0.13	0.313 ± 0.039	-1.32 ± 0.02	-2.96 ± 0.03	$+83 \pm 3$	1
G 35.2–0.7...	35.20	–0.74	0.456 ± 0.045	-0.18 ± 0.06	-3.63 ± 0.11	$+28 \pm 3$	IV
G 35.2–1.7...	35.20	–1.74	0.306 ± 0.045	-0.71 ± 0.05	-3.61 ± 0.17	$+42 \pm 3$	IV
W 51 IRS 2..	49.49	–0.37	0.195 ± 0.071	-2.49 ± 0.08	-5.51 ± 0.11	$+56 \pm 3$	III
G 59.7+0.1...	59.78	+0.06	0.463 ± 0.020	-1.65 ± 0.03	-5.12 ± 0.08	$+27 \pm 3$	III
Cep A.....	109.87	+2.11	1.430 ± 0.080	$+0.50 \pm 1.10$	-3.70 ± 0.20	-10 ± 5	II
NGC 7538....	111.54	+0.78	0.378 ± 0.017	-2.45 ± 0.03	-2.44 ± 0.06	-57 ± 3	II
IRAS 00420..	122.02	–7.07	0.470 ± 0.020	-1.99 ± 0.07	-1.62 ± 0.05	-44 ± 5	2
NGC 281.....	123.07	–6.31	0.355 ± 0.030	-2.63 ± 0.05	-1.86 ± 0.08	-31 ± 5	3
W3(OH).....	133.95	+1.06	0.512 ± 0.010	-1.20 ± 0.20	-0.15 ± 0.20	-45 ± 3	4
WB 89-437...	135.28	+2.80	0.167 ± 0.006	-1.27 ± 0.50	$+0.82 \pm 0.05$	-72 ± 3	5
S 252.....	188.95	+0.89	0.476 ± 0.006	$+0.02 \pm 0.01$	-2.02 ± 0.04	$+11 \pm 3$	I
S 269.....	196.45	–1.68	0.189 ± 0.016	-0.42 ± 0.02	-0.12 ± 0.08	$+20 \pm 3$	6
Orion.....	209.01	–19.38	2.425 ± 0.035	$+3.30 \pm 1.00$	$+0.10 \pm 1.00$	$+10 \pm 5$	7
G 232.6+1.0..	232.62	+1.00	0.596 ± 0.035	-2.17 ± 0.06	$+2.09 \pm 0.46$	$+23 \pm 3$	I
VY CMa.....	239.35	–5.06	0.876 ± 0.076	-3.24 ± 0.16	$+2.06 \pm 0.60$	$+18 \pm 3$	8

Note. — References are I: Reid et al. (2009); II: Moscadelli et al. (2009); III: Xu et al. (2009); IV: Zhang et al. (2009); V: Brunthaler et al. (2009); 1: Bartkiewicz et al. (2008); 2: Moellenbrock, Claussen & Goss (2007); 3: Sato et al. (2008); 4: Xu et al. (2006); Hachisuka et al. (2006); 5: Hachisuka et al. (2009); 6: Honma et al. (2007); 7: Hirota et al. (2007); Menten et al. (2007); 8: Choi et al. (2008). The calculations in this paper use an early parallax and proper motion estimate for IRAS 00420+5530 cited above; the values reported more recently by Moellenbrock, Claussen & Goss (2009) are slightly different but would not substantially change the results presented here.

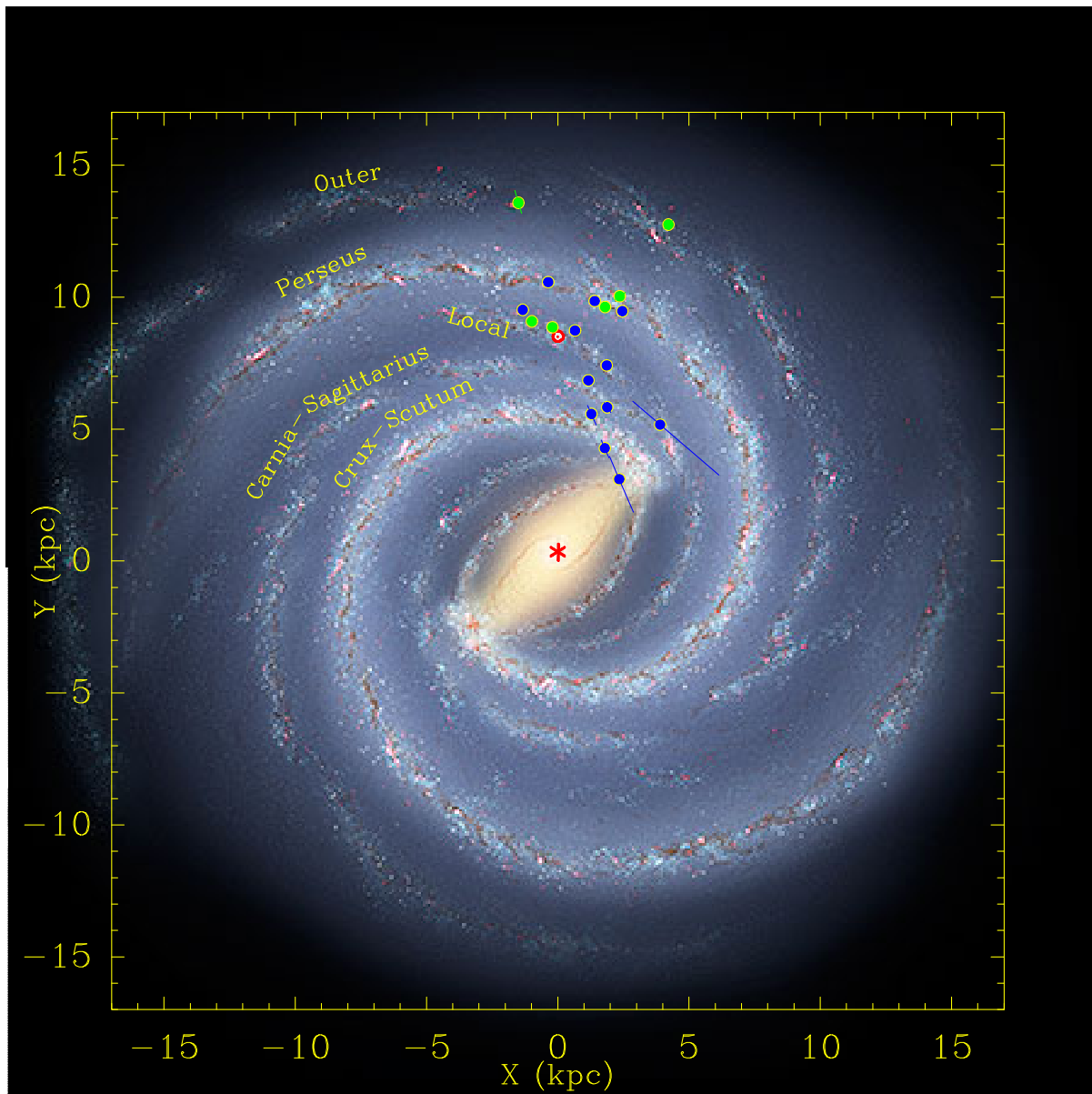


Fig. 1.— Locations of high-mass star forming regions for which trigonometric parallaxes have been measured. Parallaxes from 12 GHz methanol masers are indicated with *dark blue dots* and those from H₂O and SiO masers or continuum emission (Orion) are indicated with *light green dots*. Distance error bars are indicated, but most are smaller than the dots. The Galactic center (*red asterisk*) is at (0,0) and the Sun (*red Sun symbol*) at (0,8.5). The background is an artist's conception of Milky Way (R. Hurt: NASA/JPL-Caltech/SSC) viewed from the NGP from which the Galaxy rotates clockwise. The artist's image has been scaled to place the HMSFRs in the spiral arms, some of which are labeled.

Table 2. Spiral Arm Pitch Angles

Spiral Arm	Sources	Sep. (kpc)	$\langle \ell \rangle$ ($^\circ$)	$\langle R_p \rangle$ (kpc)	$\langle \beta \rangle$ ($^\circ$)	Pitch Angle ($^\circ$)
Perseus ...	S 252 / W3(OH)	1.9	161	10.3	3	19.5 ± 1.3
.....	S 252 / IRAS 00420	2.3	155	10.2	4	20.2 ± 2.1
.....	S 252 / NGC 7538	3.0	150	10.2	6	15.5 ± 1.9
.....	W3(OH) / NGC 7538	1.1	123	9.9	11	8.7 ± 4.7
.....	Unweighted mean					16.0 ± 3.0
Local.....	G 232.6+1.0 / Orion	1.3	221	9.2	-5	35.4 ± 0.7
.....	G 232.6+1.0 / Cep A	2.1	171	9.2	-2	23.5 ± 1.3
.....	G 232.6+1.0 / G 59.7+0.1	3.8	146	8.6	2	31.5 ± 0.5
.....	VY CMa / Cep A	1.7	175	8.9	-1	12.9 ± 1.4
.....	VY CMa / G 59.7+0.1	3.3	150	8.4	3	27.3 ± 0.4
.....	Orion / G 59.7+0.1	2.5	134	8.2	6	28.9 ± 0.5
.....	Cep A / G 59.7+0.1	1.8	85	8.2	9	38.7 ± 1.6
.....	Unweighted mean					28.3 ± 3.5

Note. — Source pair separation (Sep.) and average Galactic longitude ($\langle \ell \rangle$) are in columns 3 and 4. The average Galactocentric distances and longitudes (see Fig. 7), $\langle R_p \rangle$ and $\langle \beta \rangle$, are in columns 5 and 6. Local spiral arm pitch angles are estimated by constructing a line segment between two sources and a line tangent a Galactocentric circle intersecting the midpoint of that segment; the pitch angle is the angle between the two lines. Only source pairs with separations greater than 1 kpc and less than 4 kpc were used. NGC 281, while near the Perseus arm, is associated with a superbubble and was not included in the pitch angle determinations.

2.2. Pitch Angles

Spiral arm pitch angles can be estimated when two or more sources can be identified as members of a single arm. A simple method of estimating pitch angles is to construct line segments between sources in the same section of a spiral arm. Next, construct a line tangential to a Galactocentric circle that passes through the midpoint of this segment and determine the angle between these two lines. Clearly, this will work only for pairs of sources that are neither too closely spaced (e.g. < 1 kpc), so that measurement uncertainty or local arm irregularities dominate, or separated enough (e.g. > 4 kpc) that the (straight) line segment does not closely follow the spiral. Using this method, we have estimated pitch angles for the Perseus and Local arm. These pitch angles are tabulated in Table 2. Pitch angle uncertainties (1σ) were determined by Monte Carlo simulations of pitch angles from 10^4 trials with source distances calculated from a Gaussian distribution of parallaxes consistent with the measurements.

In Table 2, we also list the unweighted mean pitch angle and the standard error of the mean for each arm. We chose not to use a weighted mean, because the scatter among individual pitch angle estimates exceeded those expected from the formal uncertainties. This would be expected if HMSFRs do not trace a perfect logarithmic spiral pattern, as seems to be the case for the Perseus and Local arms.

The Perseus arm has an average pitch angle of $16^\circ \pm 3^\circ$ between Galactic longitude 112° and 189° . This is in the upper half of pitch angle estimates of 5° to 21° for spiral arms in the Galaxy collected by Vallée (1995). The five HMSFRs with parallaxes that trace the Local “arm” indicate a mean pitch angle of 28° . However, the Local arm is probably not a global spiral arm; instead it appears to be a short segment or spur between the Carina–Sagittarius and Perseus arms.

Two sources (S 269 & WB89–437) appear to be part of the Outer arm and formally yield a pitch angle of $2^\circ 3' \pm 4^\circ 8'$. This suggests that the Outer arm might have a smaller pitch angle than the Perseus arm. This may be of significance, but with only two sources and given their large separation of 5.8 kpc, more parallaxes are needed before reaching any firm conclusions.

For other spiral arms, we have too few parallaxes to reliably determine pitch angles. The sources that are possible members of the Carina–Sagittarius arm (G 34.2–0.7, G 34.2–1.7 & W 51 IRS 2) would formally give a wide range of pitch angles. However, because one or more sources might be associated with the Local arm, we cannot reliably estimate the pitch angle of the Carina–Sagittarius arm at this time.

3. Galactic Dynamics

Given measurements of position, parallax, proper motion and Doppler shift, one has complete three-dimensional location and velocity vectors relative to the Sun. One can then construct a model of the Milky Way and adjust the model parameters to best match the data. We model the Milky Way as a disk rotating with speed $\Theta(R) = \Theta_0 + \frac{d\Theta}{dR} (R - R_0)$, where R_0 is the distance from the Sun to the Galactic center. We started the fitting process by assuming a flat rotation curve (i.e. $\frac{d\Theta}{dR} = 0$). Later, we relaxed this assumption and solved for $\frac{d\Theta}{dR}$, followed by an investigation of other forms of the Galactic rotation curve. Since all measured motions are relative to the Sun, we need to remove the peculiar (non-circular) motion of the Sun, which is parameterized by U_\odot toward the Galactic center, V_\odot in the direction of Galactic rotation, and W_\odot towards the north Galactic pole (NGP). Table 3 summarizes these and other parameters.

We adjusted the Galactic parameters so as to best match the data to the spatial-kinematic model in a least-squares sense. For each source, we treated the measured parallax (π_s), two components of proper motion (μ_α, μ_δ), and the heliocentric velocity (v_{Helio}) as data. The observed source coordinates are known to extremely high accuracy and were treated as independent variables. We adopt the Hipparcos determination of the Solar Motion (Dehnen & Binney 1998) as definitive and generally did not vary these parameters. However, in one least-squares fit, we solved for these parameters for illustrative purposes in order to compare Solar Motion results from Hipparcos stars and our HMSFRs.

Our choice of weights for the data in the least-squares fitting process requires some comment. While the heliocentric velocity of any maser spot can be measured with very high accuracy, it may not exactly reflect the motion of the HMSFR. The internal motions of the methanol masers are generally small and cause uncertainty of $\approx 3 \text{ km s}^{-1}$ (Moscadelli et al. 2002), whereas H_2O masers can be associated with fast outflow and, if not accurately modeled, can lead to larger uncertainty in the motion of the exciting star. In addition, the Virial motion of an individual massive star (associated with the masers) with respect to the entire HMSFR is likely to be $\approx 7 \text{ km s}^{-1}$ per coordinate (e.g. for a region of mass of $\sim 3 \times 10^4 M_\odot$ and radius of $\sim 1 \text{ pc}$). Therefore, we allow for a deviation of the measured motion from the center of mass of its associated HMSFR by adding an uncertainty of 7 km s^{-1} in quadrature with the internal motion estimates (between 3 and 5 km s^{-1}).

Since the parallax data is compared to a kinematic model, we considered both the parallax measurement uncertainty and a modeling uncertainty for the kinematic distance, owing to total uncertainty in the heliocentric velocity of the associated HMSFR. These two components were added in quadrature. Similarly, the proper motion weights allowed for both measurement uncertainties and the possible deviation of the measured maser motions

Table 3. Galaxy Model Parameter Definitions

Parameter	Definition
R_0	Distance of Sun from GC
Θ_0	Rotation Speed of Galaxy at R_0
$\frac{d\Theta}{dR}$	Derivative of Θ with R : $\Theta(R) = \Theta_0 + \frac{d\Theta}{dR} (R - R_0)$
U_\odot	Solar motion toward GC
V_\odot	Solar motion in direction of Galactic rotation
W_\odot	Solar motion toward NGP
\overline{U}_s	Average source peculiar motion toward GC
\overline{V}_s	Average source peculiar motion in direction of Galactic rotation
\overline{W}_s	Average source peculiar motion toward NGP

Note. — GC is the Galactic Center and NGP is the North Galactic Pole. The average source peculiar motions ($\overline{U}_s, \overline{V}_s, \overline{W}_s$) are defined at the location of the source and are rotated with respect to the solar motion ($U_\odot, V_\odot, W_\odot$) by the Galactocentric longitude, β , of the source (see Appendix Fig. 7). We solve for the magnitude of each component, but the orientation of the vector for each source depends on location in the Galaxy.

from the center of mass of the HMSFR. The latter term was set by the uncertainty in the heliocentric velocity divided by the distance.

3.1. Galactic 3-D Motions

We first used all 18 sources listed in Table 1 and solved only for the fundamental Galactic parameters, yielding $R_0 = 8.2$ kpc and $\Theta_0 = 265$ km s⁻¹ for a flat rotation curve ($\frac{d\Theta}{dR} = 0$) (see Fit 1 in Table 4). The χ^2 value of 263 for 70 degrees of freedom was quite large. The residuals to the fit showed clear systematic deviations, with almost all sources exhibiting large residual motions counter to Galactic rotation and some toward the Galactic center.

Fig. 2 shows the peculiar motions of the HMSFRs in the Galactic plane by transforming to a reference frame that rotates with the Galaxy, calculated with the IAU recommended values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s⁻¹ and assuming a flat rotation curve. (These calculations are documented in the Appendix.) Sizeable systematic motions are clearly evident. Almost all sources have a significant component of peculiar motion *counter* to Galactic rotation. On average these star forming regions orbit the Galaxy ≈ 15 km s⁻¹ slower than the Galaxy spins. This conclusion is not sensitive to the values adopted for R_0 or Θ_0 and would not change qualitatively were we to increase the rotation speed of the Galaxy to $\Theta_0 = 251$ km s⁻¹, a value consistent with the proper motion of Sgr A* (Reid & Brunthaler 2004). Similarly, adopting a more complex rotation curve, e.g. the Clemens (1985) curve for $R_0 = 10$ kpc and $\Theta_0 = 250$ km s⁻¹ with distances scaled to 85% to give $R_0 = 8.5$ kpc, does not change the qualitative nature of the residuals. *HMSFRs appear to orbit the Galaxy slower than for circular orbits.* This might be explained by star formation triggered by the encounter of molecular gas with a shock front associated with a trailing spiral arm and may help explain the 17 km s⁻¹ dispersion seen in HI data by Brand & Blitz (1993).

For the distribution of sources in our sample, the solar motion parameters U_\odot and V_\odot can partially mimic the average source peculiar motions. We believe the solar motion parameters determined from Hipparcos data by Dehnen & Binney (1998) are well determined, and they have been independently confirmed by Méndez et al. (1999), based on the Southern Proper-Motion program data. However, it is instructive to solve for the Solar Motion parameters with the maser data. Doing so we find an acceptable fit with $U_\odot = 9$ km s⁻¹, $V_\odot = 20$ km s⁻¹ and $W_\odot = 10$ km s⁻¹. (The χ^2 value for this fit was 67.2 for 59 degrees of freedom, which is somewhat worse than the value of 65.7 found in §3.2, where we adopt the Hipparcos solar motion parameters and solved instead for average source peculiar motion components.)

In Fig. 3 we reproduce the Hipparcos solar motion data from Fig. 4 of Dehnen & Binney

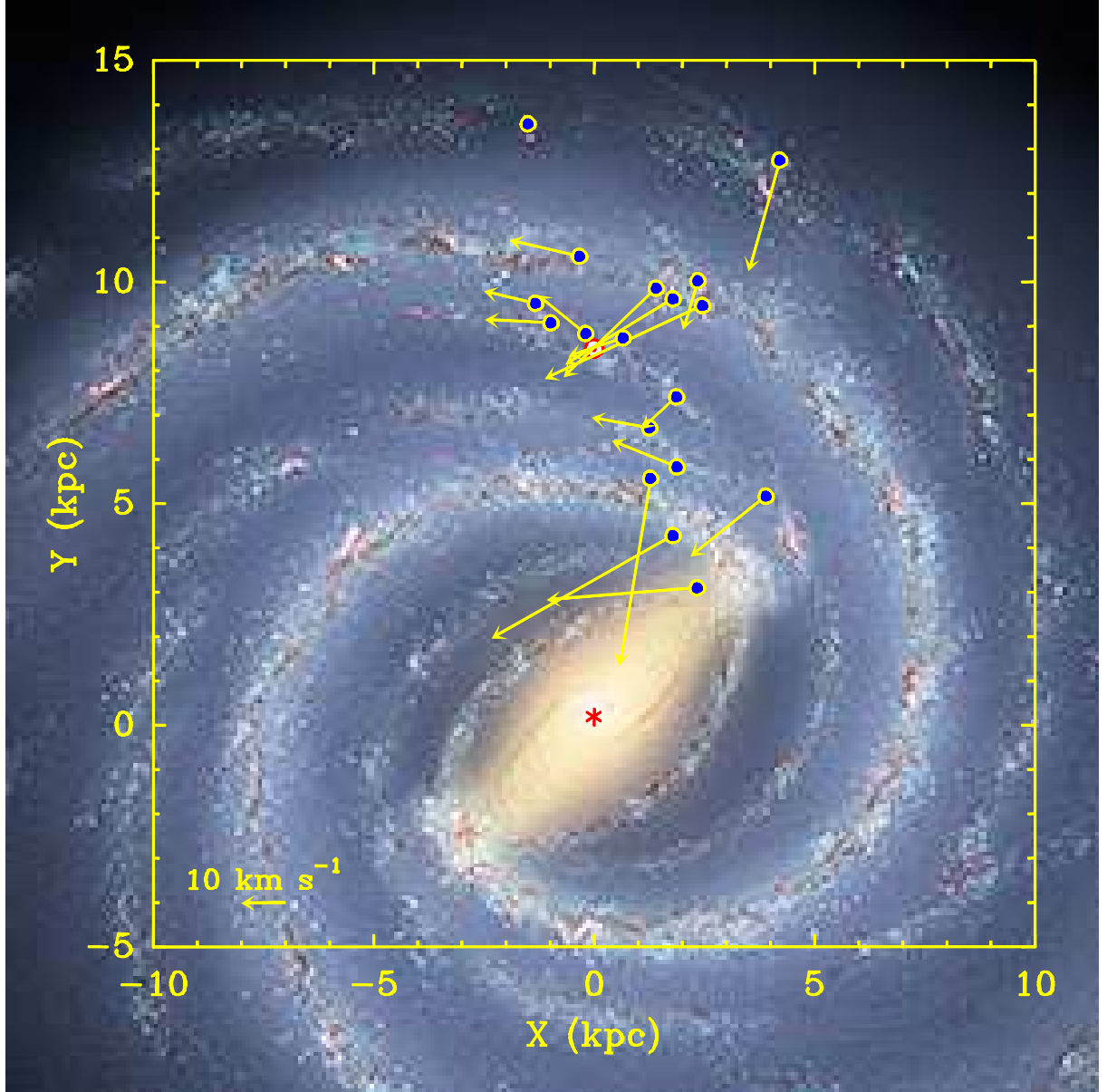


Fig. 2.— Peculiar motion vectors of high mass star forming regions (superposed on an artist conception) projected on the Galactic plane after transforming to a reference frame rotating with the Galaxy, using IAU standard values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s $^{-1}$ and a flat rotation curve. A 10 km s $^{-1}$ motion scale is in the lower left. The Galaxy is viewed from the north Galactic pole and rotates clockwise.

(1998). Their data were binned by stellar colors, plotted against stellar dispersion and the solar motion components estimated as minus the average velocity of all stars in each bin. We have also plotted our estimates of the solar motion, plotted at near-zero “stellar dispersion” appropriate for newly formed stars. Also included in the bottom panel is the value of W_{\odot} determined from the proper motion of Sgr A* by Reid & Brunthaler (2004), assuming that the supermassive black hole is stationary at the Galactic center. These values for U_{\odot} and W_{\odot} are in good agreement with the Hipparcos results.

The Hipparcos data for V_{\odot} (the solar motion component in the direction of Galactic rotation) clearly show the well known “asymmetrical drift,” which when extrapolated to zero dispersion should define the LSR. When we solve for the solar motion with the HMSFR parallax and proper motions, we find a value of V_{\odot} of 20 km s^{-1} . This is far above the asymmetric drift line, indicating that the HMSFRs as a group are orbiting the Galaxy slower than for circular orbits. Note that the youngest stars in the Hipparcos data plotted at a dispersion of $\approx 120 \text{ (km s}^{-1})^2$ show a similar, but not as great a departure from the asymmetrical drift line. Evidence that young stars lag the LSR orbit has also been found by Zabolotskikh, Rastorguev & Dambis (2002).

Finally, we note that we find no evidence for a global motion of the LSR (i.e. disagreement with the Hipparcos Solar Motion) in the direction of Galactic center or out of the plane of the Galaxy larger than 6 km s^{-1} (2σ). This is contrary to the conclusions of Kerr (1962) and Blitz & Spergel (1991a), based on an analysis of HI data, that the LSR is moving away from the Galactic center at a speed of $> 10 \text{ km s}^{-1}$.

3.2. Fundamental Galactic Parameters

Since, as shown in §3.1, HMSFRs are orbiting the Galaxy slower than for circular orbits, we must allow for such effects when modeling the Galaxy. In order to determine the fundamental parameters R_0 and Θ_0 , we solved for three additional parameters allowing for an average peculiar motion for all sources with components \overline{U}_s toward the Galactic center (as seen by the source), \overline{V}_s in the local direction of Galactic rotation and \overline{W}_s toward the north Galactic pole. This solution, listed as Fit 2 in Table 4, yielded comparable values of $R_0 = 8.5 \text{ kpc}$ and $\Theta_0 = 264 \text{ km s}^{-1}$ and peculiar motion components of $\overline{U}_s = 4 \text{ km s}^{-1}$, $\overline{V}_s = -16 \text{ km s}^{-1}$ and $\overline{W}_s = 3 \text{ km s}^{-1}$. The residuals showed greatly reduced systematic deviations, and the χ^2 value improved significantly to 112 for 67 degrees of freedom, compared to the solution without the average peculiar motions (Fit 1 in Table 4).

Two sources from the sample, NGC 7538 and G 23.6–0.1, had post-fit residuals that

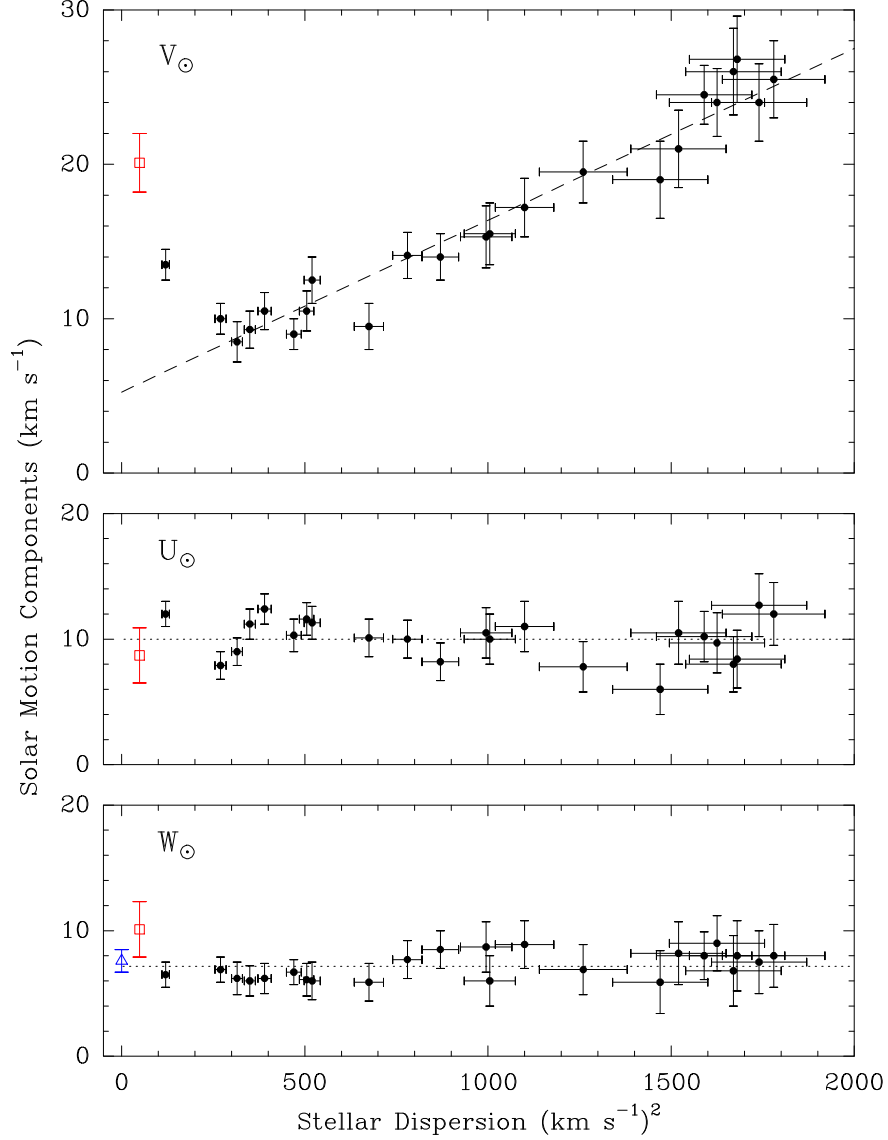


Fig. 3.— Solar motion components determined from Hipparcos stars (i.e. the reflex of the average motion of stars) versus stellar velocity dispersion after Dehnen & Binney (1998). *Top Panel:* V_{\odot} is the Solar Motion in the direction of Galactic rotation (i.e. toward $\ell = 90^{\circ}$). The “asymmetrical drift” is shown with the *dashed line*. *Middle Panel:* U_{\odot} is the Solar Motion toward the Galactic center. *Bottom Panel:* W_{\odot} is toward the north Galactic pole. Also plotted at 50 (km s⁻¹)² dispersion with *open red squares* are solar motion parameters obtained from the parallax and proper motions of star forming regions, and at zero dispersion with an *open triangle* is the W_{\odot} component inferred from the proper motion of Sgr A* by Reid & Brunthaler (2004). Note the good agreement of the U_{\odot} and W_{\odot} components between Hipparcos and this study. The large deviation of the V_{\odot} component from the asymmetrical drift from this study is not indicative of large V_{\odot} value, but points to a significant deviation from circular orbits for very young stars.

were significantly greater ($> 3\sigma$) than the others. Removing these sources, we arrive at our “basic sample” of 16 HMSFRs. We repeated the fitting and found $R_0 = 8.40 \pm 0.36$ kpc, $\Theta_0 = 254 \pm 16$ km s $^{-1}$, $\overline{U}_s = 2.3 \pm 2.1$ km s $^{-1}$, $\overline{V}_s = -14.7 \pm 1.8$ km s $^{-1}$ and $\overline{W}_s = 3.0 \pm 2.1$ km s $^{-1}$ (see Fit 3 in Table 4). The χ^2 value for this sample was considerably improved: 65.7 for 59 degrees of freedom. The near-zero average motion out of the plane of the Galaxy is as expected for massive star forming regions. The residual motions in the plane of the Galaxy are shown in Fig. 4. Most of the star forming regions have residual velocities consistent with measurement error combined with expected Virial motions within HMSFRs of ~ 7 km s $^{-1}$ per coordinate. The most distant sources at low Declination (and low Galactic longitude) have larger residual velocities owing to greater parallax and proper motion measurement uncertainty and the scaling of proper motions to linear speeds by multiplying by distance.

We consider that this solution provides the best estimates of the parameters for the current data set, under the assumption of a flat rotation curve. In §3.3 we show that the estimate of R_0 is somewhat sensitive to the nature of the rotation curve of the Galaxy, leading to a systematic source of uncertainty for R_0 of approximately ± 0.5 kpc. Combining the statistical and systematic uncertainties in quadrature, we find $R_0 = 8.4 \pm 0.6$ kpc.

The correlation coefficient between R_0 and Θ_0 was 0.87, while all others were small. This is expected, since kinematic model distances increase with R_0 and inversely with Θ_0 . Thus, the ratio Θ_0/R_0 , which is the angular rotation rate of the LSR, is determined to much better accuracy than either parameter separately. Holding $R_0 = 8.50$ kpc (the IAU recommended value), we find $\Theta_0 = 257.9 \pm 7.7$ km s $^{-1}$ or $\Theta_0/R_0 = 30.3 \pm 0.9$ km s $^{-1}$ kpc $^{-1}$. There is only a slight dependence of Θ_0/R_0 on the value adopted for R_0 . For example, setting $R_0 = 8.00$ kpc, we obtain $\Theta_0/R_0 = 30.0 \pm 0.9$ km s $^{-1}$ kpc $^{-1}$. See §6 for a discussion of the significance of this result.

As shown in §3.3, while estimates of Θ_0 change by ± 20 km s $^{-1}$ among the fits using different rotation curves, this variation can be accounted for mostly through the correlation with R_0 , and, therefore, the least-squares fitting process incorporates this correlation in the formal uncertainty estimate. Thus, we conclude that the formal uncertainty of ± 16 for Θ_0 is a reasonable (provided that R_0 is within 0.5 kpc of 8.4 kpc). When R_0 is ultimately measured with much higher accuracy, Θ_0 would be even better determined from the well determined ratio of Θ_0/R_0 .

We also considered the possibility that a large positive value for \overline{U}_s (toward the Galactic center), as could be expected from spiral density wave theory, might inflate the estimate of Θ_0 . Holding $\overline{U}_s = 17$ km s $^{-1}$ (15 km s $^{-1}$ greater than our best fit) did not significantly reduce the estimate of Θ_0 , but did dramatically increase the χ^2 to 200.1. Thus, we exclude such a large \overline{U}_s value and that it could contribute to significant uncertainty in Θ_0 .

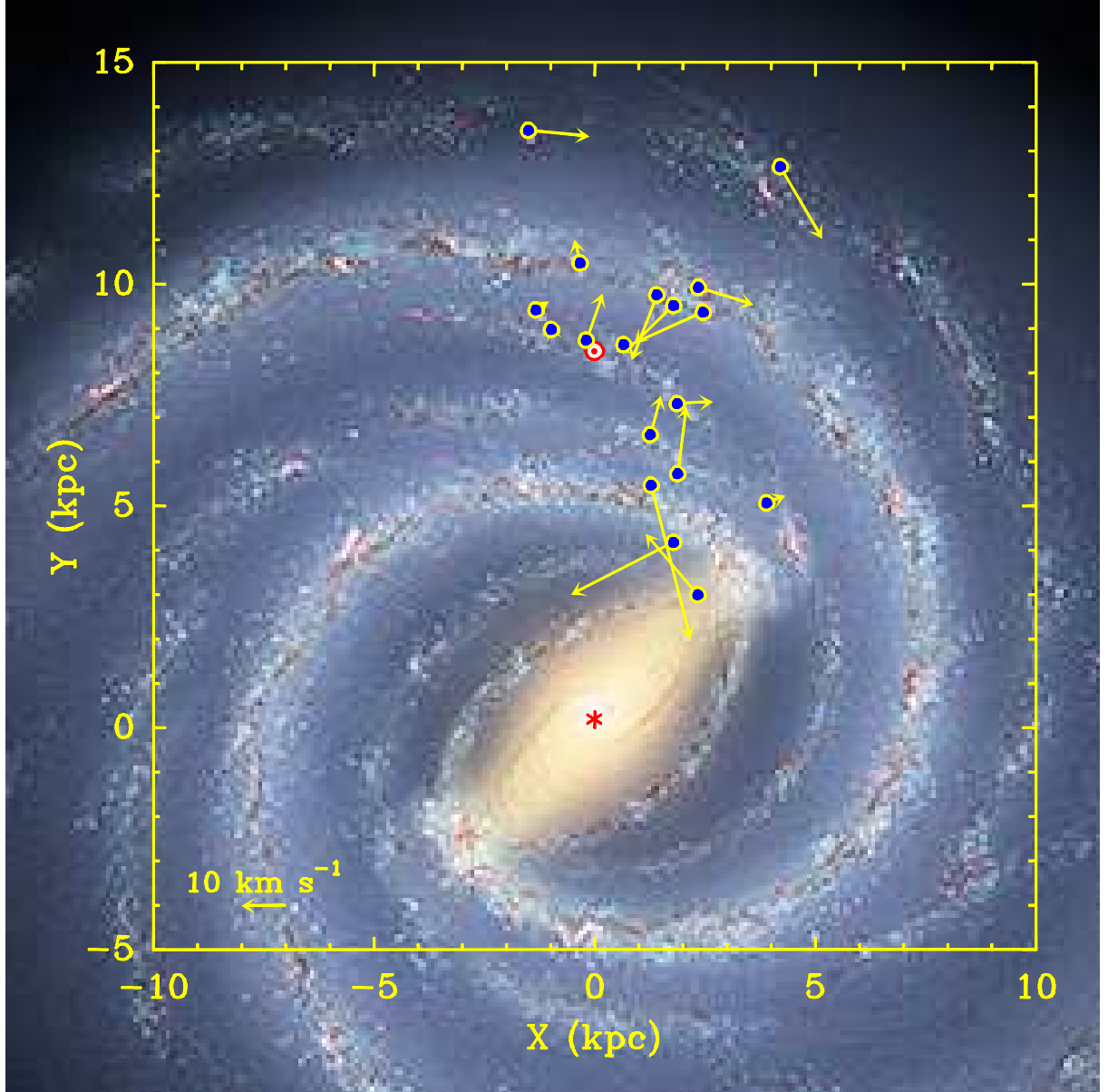


Fig. 4.— Peculiar motion vectors of high mass star forming regions (superposed on an artist conception) after transforming to a reference frame rotating with the Galaxy, using best-fit values of $R_0 = 8.4$ kpc and $\Theta_0 = 254$ km s $^{-1}$ and removing an average motion of 15 km s $^{-1}$ counter to Galactic rotation and 2 km s $^{-1}$ toward the Galactic center. A 10 km s $^{-1}$ motion scale is in the lower left. The Galaxy is viewed from the north Galactic pole and rotates clockwise.

Table 4. Least-squares Fitting Results

Fit	R_0 (kpc)	Θ_0 (km s ⁻¹)	$\frac{d\Theta}{dR}$ (km s ⁻¹ kpc ⁻¹)	\overline{U}_s (km s ⁻¹)	\overline{V}_s (km s ⁻¹)	\overline{W}_s (km s ⁻¹)	χ^2	DF	Θ_0/R_0 (km s ⁻¹ kpc ⁻¹)
1	8.24±0.55	265±26	0.0	0.0	0.0	0.0	263.3	70	32.4±1.3
2	8.50±0.44	264±19	0.0	3.9±2.5	-15.9±2.1	3.1±2.5	111.5	67	31.1±1.1
3	8.40±0.36	254±16	0.0	2.3±2.1	-14.7±1.8	3.0±2.2	66.7	59	30.3±0.9
4	9.04±0.44	287±19	2.3±0.9	1.9±2.0	-15.5±1.7	3.0±2.1	59.0	58	31.1±0.9
5	8.73±0.37	272±15	Clemens-10	1.7±1.9	-12.2±1.7	3.1±1.9	52.9	59	31.0±0.8
6	7.88±0.30	230±12	Clemens-8.5	2.7±2.2	-12.4±1.9	3.1±2.3	71.2	59	29.6±1.0
7	8.79±0.33	275±13	Brand-Blitz	1.9±2.0	-18.9±1.8	3.0±2.1	59.0	59	31.0±0.9

Note. — Fits 1 & 2 used all 18 sources in Table 1 and have high χ^2 values, owing to two outliers: NGC 7538 and G 23.6−0.1. Fit 3 excludes the two outliers and provides our basic result, under the assumption of a flat rotation curve. Fits 4 – 7 explore the effects of non-flat rotation curves. “DF” is the degrees of freedom for the fit (i.e. number of data equations minus number of parameters). $(\overline{U}_s, \overline{V}_s, \overline{W}_s)$ are average peculiar motions common to all sources (see Table 7 and Fig. 7), assuming the Hipparcos solar motion of Dehnen & Binney (1998) (see discussion in §3.1). All Θ_0/R_0 estimates were obtained by holding $R_0 = 8.50$ kpc and solving for Θ_0 . “Clemens-10” and “Clemens-8.5” refer to the Clemens (1985) rotation curves for $(R_0[\text{kpc}], \Theta_0[\text{km s}^{-1}]) = (10, 250)$ and $(8.5, 220)$, respectively; “Brand-Blitz” refers to the Brand & Blitz (1993) rotation curve. Both the Clemens and Brand-Blitz rotation curves were scaled to the fitted values of R_0 and Θ_0 .

3.3. Rotation Curves

We have until now assumed that the rotation curve of the Galaxy is flat (i.e. $\Theta(R) = \Theta_0$). In order to investigate deviations from a flat rotation curve, we used the basic sample of 16 sources and added the parameter $\frac{d\Theta}{dR}$ to the model. A least-squares fit yielded $\frac{d\Theta}{dR} = 2.3 \pm 0.9 \text{ km s}^{-1} \text{ kpc}^{-1}$, with an improved χ^2 compared to the flat rotation curve fit (see Fit 4 in Table 4), but with an increased correlation coefficient between R_0 and Θ_0 of 0.90. We found that the estimate of Θ_0 was insensitive to whether R_0 was solved for ($R_0 = 9.0 \text{ kpc}$) or held at 8.5 kpc (to reduce correlations). We tested how sensitive $\frac{d\Theta}{dR}$ was to the two outer Galaxy sources by dropping S 269 and WB 89-437 from the sample and re-fitting. This yielded $\frac{d\Theta}{dR} = 1.9 \pm 1.2 \text{ km s}^{-1} \text{ kpc}^{-1}$ and indicated that these sources do not provide all the leverage for a rising rotation curve. Thus, we find a nearly flat rotation curve between radii of about 4 to 13 kpc, with some evidence for a slight rise with distance from the Galactic center. This supports similar conclusions reached in a number of papers (Fich, Blitz & Stark 1989; Brand & Blitz 1993; Honma & Sofue 1997; Maciel & Lago 2005). For example Fich, Blitz & Stark (1989) find that the rotation curve is nearly flat for $\Theta_0 = 220 \text{ km s}^{-1}$ and that it rises gradually for $\Theta_0 = 250 \text{ km s}^{-1}$.

We also tested more complex rotation curves by replacing the simple linear form just discussed with the rotation curves of Clemens (1985) and Brand & Blitz (1993). Clemens supplied two curves: one assuming the old IAU constants of $R_0 = 10 \text{ kpc}$ and $\Theta_0 = 250 \text{ km s}^{-1}$ and the other assuming the new constant of $R_0 = 8.5 \text{ kpc}$ and $\Theta_0 = 220 \text{ km s}^{-1}$. These models have slightly different shapes, with the old model generally having rotational speeds that rise faster with radius than the new model. For either model, we fitted for different values of R_0 (which we used to scale model radii) and Θ_0 (which we used to scale rotation speeds). The fit using the old model, listed as Fit 5 in Table 4, gave $R_0 = 8.7 \pm 0.4 \text{ kpc}$ and $\Theta_0 = 272 \pm 15 \text{ km s}^{-1}$, with an improved $\chi^2 = 52.9$ for 59 degrees of freedom compared to our solution for a flat rotation curve. The improvement is partly from a better match to the two sources in the Outer arm (S 269 and WB 89-437). Using the new rotation model, gave $R_0 = 7.9 \pm 0.3 \text{ kpc}$ and $\Theta_0 = 230 \pm 12 \text{ km s}^{-1}$, with a considerably worse $\chi^2 = 71.2$ (see Fit 6 in Table 4). Using the Brand & Blitz (1993) rotation curve, also scaled by the fitted values of R_0 and Θ_0 , we obtain Fit 7 in Table 4, with values of $R_0 = 8.8 \pm 0.4 \text{ kpc}$ and $\Theta_0 = 275 \pm 15 \text{ km s}^{-1}$ and a $\chi^2 = 59.0$ for 59 degrees of freedom, intermediate between the χ^2 values for the two Clemens models.

Clearly there is some sensitivity of the best fit R_0 value to the models, and we adopt a systematic uncertainty in R_0 of $\pm 0.5 \text{ kpc}$. Note that, as discussed in §3.2, the ratio Θ_0/R_0 has much less modeling sensitivity. With the current parallax and proper motion data, we cannot conclusively distinguish among the rotation curves presented. However, with the

many more parallaxes and proper motions expected in the next few years from the VLBA and VERA arrays, we should be able to make considerable progress in refining the rotation curve of the Milky Way.

4. Kinematic Distances

Fig. 5 compares the locations of the star forming regions determined by trigonometric parallax and by kinematic distances. The kinematic distances were computed for the IAU standards $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s⁻¹ and the standard definition of LSR. For 13 of 18 regions (11 of 16 in the basic sample), the kinematic distance exceeds the true source distance; in 3 cases the discrepancy is over a factor of two. The kinematic distances for (these) star forming regions tend to over-estimate the source distances.

As shown above, HMSFRs on average orbit the Galaxy ≈ 15 km s⁻¹ slower than the circular rotation speed. Taking this into account, a prescription for a “revised” kinematic distance for a high-mass star forming region could be as follows:

- 1) add back the (old) Standard Solar Motion corrections to the LSR velocities, returning them to the heliocentric frame;
- 2) apply “best values” for the solar motion to calculate a revised “LSR” velocity, v_{LSR}^r ;
- 3) subtract -15 km s⁻¹ from the velocity component in the direction of Galactic rotation;
- 4) calculate a kinematic distance using values for the fundamental parameters of the Milky Way, e.g. $R_0 = 8.4$ kpc and $\Theta_0 = 254$ km s⁻¹, that are consistent with astrometric measurements; and
- 5) when determining the *uncertainty* in the kinematic distance, include a systematic contribution allowing for the possibility of a 7 km s⁻¹ uncertainty in v_{LSR}^r .

Table 5 gives parallax distances, standard (old) kinematic distances, and revised kinematic distances and uncertainties (using the above prescription) for all 18 HMSFRs listed in Table 1. (We provide the FORTRAN source code used to calculate revised kinematic distances in on-line material.) Note that our prescription for the uncertainty in kinematic distances performs reasonably well for our basic sample (excluding the two sources G 23.6–0.1 and NGC 7538 which we earlier noted as outliers). The mean difference between the parallax and kinematic distances is near zero and the differences divided by their uncertainties average to near unity. Now only roughly half (9 of 16) of the sources in the basic sample have

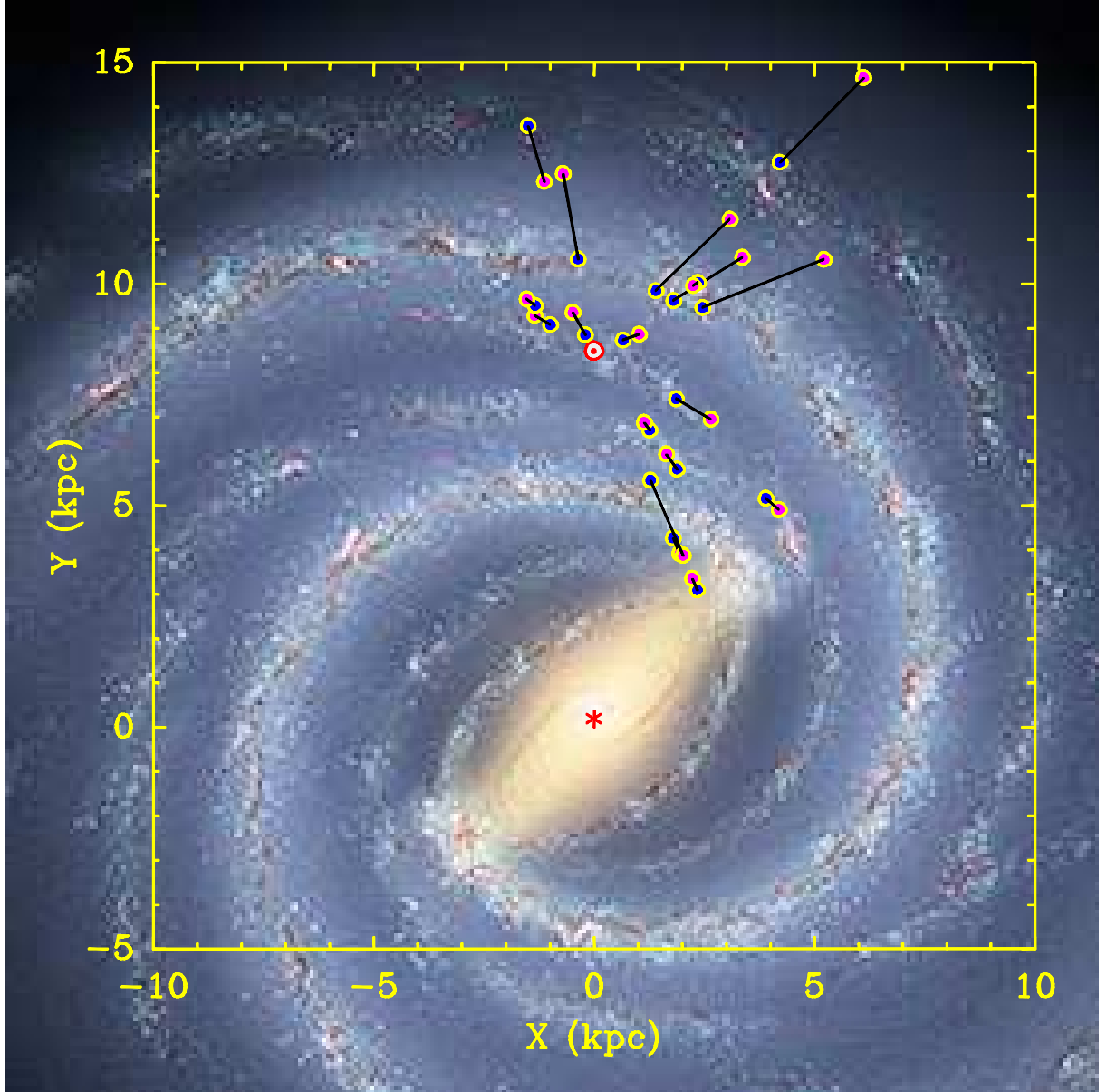


Fig. 5.— The locations of the star forming regions determined by trigonometric parallax (*dark blue circles*) and by kinematic distances (*light magenta circles*), assuming IAU recommended values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s $^{-1}$ and the Standard Solar Motion to define the LSR.

kinematic distances that exceed the true source distance. Only in one case (Orion) is the discrepancy is over a factor of two, and the estimated uncertainty accounts for discrepancy.

While the prescription outlined above results in some improvement in kinematic distances compared to the standard approach, the improvement is not as great as one might at first expect. This occurs because the (standard) definition of v_{LSR} uses the Standard Solar Motion. While the Standard Solar Motion differs only slightly from the Hipparcos solar motion for components toward the Galactic center (U_{\odot}) and the north Galactic pole (W_{\odot}), there is a large discrepancy for the component in the direction of Galactic rotation. The Standard value is $V_{\odot}^{\text{Std}} = 15.3 \text{ km s}^{-1}$, whereas the Hipparcos value is $V_{\odot}^H = 5.25 \text{ km s}^{-1}$. The $+10 \text{ km s}^{-1}$ “error” in the Standard V_{\odot}^{Std} partially compensates for the 15 km s^{-1} slower Galactic orbits of HMSFRs shown in §3.1. (Note that a positive change in the solar motion component V_{\odot} results in a negative change in a source peculiar motion component \bar{V}_s). Even with the improved prescription for kinematic distances, one cannot really hope to discern spiral structure using kinematic distances.

5. Galactic Coordinates

There is excellent agreement between the two independent VLBI measurements of Θ_0/R_0 : the measurement based on parallaxes and proper motions of HMSFRs (this paper) and on the proper motion of Sgr A* (Reid & Brunthaler 2004). This gives us confidence that 1) we can well model the Galaxy with parallax and proper motions of HMSFRs and 2) Sgr A* is indeed a supermassive black hole at the dynamical center of the Milky Way. These findings offer a possible new definition of the Galactic plane and Galactic coordinates. Currently, the IAU definition of the Galactic plane is based primarily on the thin distribution of neutral hydrogen 21 cm emission (Blaauw et al. 1960). The Sun was defined to be precisely in the plane (i.e. at $b = 0$) and the origin of longitude was set by the centroid of the radio emission of the large, complex source Sgr A. The Sun is now known to be $\approx 20 \text{ pc}$ north of the plane (see Reid (2006) and references therein) and the supermassive black hole, Sgr A*, is offset by a few arcmin from the IAU defined center.

One could redefine Galactic coordinates based on the proper motion of Sgr A*, which, after correction for the well-determined solar motion component perpendicular to the Galactic plane, gives the orbital plane of the Local Standard of Rest. The zero of longitude would be best defined by the position of Sgr A*. This would place our supermassive black hole at the origin of Galactic coordinates and would remove the Sun from its special location precisely in the Galactic plane. It is worth noting that with such a new definition, Galactic coordinates would be time variable as the Sun orbits the Milky Way.

Table 5. Parallaxes vs. Kinematic Distances

Source	l (deg)	b (deg)	v_{LSR} km s ⁻¹	D_{π} (kpc)	D_k^{Std} (kpc)	D_k^{Rev} (kpc)
G 23.0–0.4...	23.01	–0.41	+81	4.59	4.97	$4.72^{+0.3}_{-0.3}$
G 23.4–0.2...	23.44	–0.18	+97	5.88	5.60	$5.30^{+0.3}_{-0.3}$
G 23.6–0.1...	23.66	–0.13	+83	3.19	5.04	$4.77^{+0.3}_{-0.3}$
G 35.2–0.7...	35.20	–0.74	+28	2.19	1.98	$1.98^{+0.4}_{-0.4}$
G 35.2–1.7...	35.20	–1.74	+42	3.27	2.85	$2.78^{+0.4}_{-0.4}$
W 51 IRS 2..	49.49	–0.37	+56	5.13	5.52	$5.46^{+1.6}_{-1.6}$
G 59.7+0.1...	59.78	+0.06	+27	2.16	3.07	$3.45^{+0.7}_{-1.2}$
Cep A.....	109.87	+2.11	–10	0.70	1.09	$0.58^{+0.7}_{-0.7}$
NGC 7538....	111.54	+0.78	–57	2.65	5.61	$4.65^{+0.7}_{-0.6}$
IRAS 00420..	122.02	–7.07	–44	2.13	3.97	$3.10^{+0.6}_{-0.6}$
NGC 281.....	123.07	–6.31	–31	2.82	2.69	$2.01^{+0.6}_{-0.6}$
W3(OH).....	133.95	+1.06	–45	1.95	4.28	$3.43^{+0.7}_{-0.7}$
WB 89-437...	135.28	+2.80	–72	5.99	8.68	$6.94^{+1.2}_{-1.0}$
S 252.....	188.95	+0.89	+11	2.10	4.06	$3.28^{+4.1}_{-2.4}$
S 269.....	196.45	–1.68	+20	5.29	3.98	$3.28^{+2.0}_{-1.5}$
Orion.....	209.01	–19.38	+10	0.41	0.99	$0.90^{+0.7}_{-0.6}$
G 232.6+1.0..	232.62	+1.00	+23	1.68	1.92	$1.43^{+0.6}_{-0.5}$
VY CMa.....	239.35	–5.06	+18	1.14	1.56	$1.15^{+0.6}_{-0.6}$

Note. — D_{π} is the measured parallax converted to distance; D_k^{Std} is the kinematic distance based on standard LSR velocities; D_k^{Rev} and $\sigma(D_k)$ is the revised kinematic distance and its uncertainty, calculated for $R_0 = 8.4$ kpc, $\Theta_0 = 254$ km s⁻¹ and $\overline{U_s} = -15$ km s⁻¹, following the prescription outlined in §4. All kinematic distances assume a flat rotation curve.

6. Discussion

Very Long Baseline Interferometry now routinely yields parallax measurements with accuracies of $\sim 10 \mu\text{as}$, corresponding to 10% uncertainty at a distance of 10 kpc, and proper motions that are usually accurate to better than a few km s^{-1} at similar distances. Target sources include molecular masers associated with star formation and red giant stars, as well as non-thermal continuum emission associated with young, low-mass (e.g. T Tau) stars and cool dwarf stars. Combining the first results of parallaxes for high-mass star forming regions from the VLBA and the Japanese VERA project has allowed us to begin to investigate the spiral structure and kinematics of the Galaxy.

We have accurately located three of the spiral arms of the Milky Way and directly measured a pitch angle of 16° for a portion of the Perseus spiral arm. This pitch angle is similar to those of spiral arms in other galaxies of type Sb to Sc (Kennicutt 1981). Two armed spirals can account for most of the known large H II regions only if the arms wrap twice around the Galaxy; this requires pitch angles of $\approx 8^\circ$. With a pitch angles greater than $\approx 12^\circ$, the Galaxy needs to have four arms in order to account for the approximate locations of H II regions (Georgelin & Georgelin 1976; Taylor & Cordes 1993). There has been considerable discussion in the literature concerning then number of spiral arms in the Galaxy (Simonson 1976; Bash 1981; Vallée 1995; Drimmel 2000; Drimmel & Spergel 2001; Benjamin et al. 2005; Nakanishi & Sofue 2006; Steiman-Cameron, Wolfire & Hollenbach 2008), with Spitzer GLIMPSE survey results suggesting only two arms can be traced in the redder, older population of stars (Benjamin 2008). Perhaps the VLBI and infrared survey results can be reconciled if the Milky Way exhibits a hybrid structure, consisting of two dominant spiral arms, populated by both young and old stars and with pitch angles near 16° , and two weaker arms traced only by young stars.

Our finding that HMSFRs on average orbit the Galaxy $\approx 15 \text{ km s}^{-1}$ slower than expected for circular orbits has implications for star formation and spiral density wave theory. The plot of the *apparent* solar motion in the direction of Galactic rotation (V_\odot) versus stellar dispersion (Fig. 3) can be interpreted as a time sequence, with stellar age increasing with dispersion. The 15 km s^{-1} slower orbital speed of HMSFRs displays as a positive departure of the apparent Solar Motion with respect to the asymmetric drift, since the Sun *appears* to orbit faster when measured against such stars. One explanation for this finding is that HMSFRs are born in elliptical Galactic orbits, near apocenter, with orbital eccentricity of about 0.06. As young stars continue to orbit the Galaxy, their orbits become more circularized, as evidenced by the lesser departure of the youngest Hipparcos bin (mostly late B-type stars) from the asymmetric drift line compared to the HMSFRs. The gradual transfer of angular momentum from gas to stars in the Galaxy proposed by Chakrabarti

(2009) may explain this. At a stellar dispersion of $\approx 300 \text{ (km s}^{-1})^2$, which corresponds to A2- to A5-type stars with colors $B - V = 0.1$ and characteristic main-sequence lifetimes of $\sim 1 \text{ Gy}$, the stars join the asymmetric drift. As stars continue to age, their orbits are progressively “randomized” and they (again) become part of a slower orbiting population, which appears as a larger apparent V_\odot .

Parallaxes measurements alone generally cannot yield R_0 (except for a parallax of Sgr A*). One needs a model of the Galaxy to compare with the measured distances in order to determine the scale of the Galaxy. Since galaxies rotate in a fairly smooth fashion, a kinematic model can be directly compared with distance and relative motion measurements in order to estimate R_0 and Θ_0 . In this paper, we have demonstrated that parallax and proper motion measurements to HMSFRs across large portions of the Galaxy can separate estimates of R_0 and Θ_0 , although because of the somewhat restricted coverage of the Galaxy currently available, we currently have a significant correlation between these parameters. Our best estimate of R_0 is $8.4 \pm 0.36 \pm 0.5 \text{ kpc}$, where second uncertainty is systematic and comes from our lack of detailed knowledge of the rotation curve of the Galaxy. This estimate is consistent with the “best” R_0 of $8.0 \pm 0.5 \text{ kpc}$ from a combination of many methods reviewed by Reid (1993). Also, recent *direct* estimates of R_0 from radial velocities and elliptical paths of stars that orbit Sgr A* have converged on values of $8.4 \pm 0.4 \text{ kpc}$ (Ghez et al. 2008) and 8.33 ± 0.35 (Gillessen et al. 2009). (These estimates assume that Sgr A* is nearly motionless at the Galactic center. Relaxing this assumption decreases the the estimates to about 8.0 kpc .) Of course, many other less direct estimates of R_0 can be found in the literature and span a much greater range.

The characteristic rotation speed of the Galaxy (Θ_0) is a crucial parameter not only for Galactic dynamics and kinematic distance determinations, but also for estimating the total mass in dark matter and the history and fate of the Local Group of galaxies (Shattow & Loeb 2008; Loeb et al. 2005). Estimates of the rotation speed of the Galaxy from the recent literature span a very large range between 184 km s^{-1} (Olling & Merrifield 1998) and 272 km s^{-1} (Méndez et al. 1999). Most estimates of Θ_0 are based on analyses of the shear and rotation of large samples of stars in the (extended) solar neighborhood and thus really measure Oort’s A and B parameters. Quoted values of Θ_0 then come by *assuming* a value for R_0 and using the relation $\Theta_0 = R_0(A - B)$. Our result that $\Theta_0 = 254 \pm 16 \text{ km s}^{-1}$ was obtained by fitting for both R_0 and Θ_0 using full 3-dimensional locations and motions of sources well beyond the extended solar neighborhood and, thus, does not assume a value for R_0 . However, as discussed in §3.2, with the present distribution of sources there is considerable correlation between R_0 and Θ_0 parameters, which is reflected in the $\pm 16 \text{ km s}^{-1}$ formal uncertainty for Θ_0 .

Our estimate of the ratio Θ_0/R_0 of $30.3 \pm 0.9 \text{ km s}^{-1} \text{ kpc}^{-1}$ is determined more accurately than either parameter individually and is nearly independent of the value of R_0 over the range of likely values between about 8.0 and 8.5 kpc. This value differs considerably from that determined from the IAU values of $\Theta_0/R_0 = 220 \text{ km s}^{-1}/8.5 \text{ kpc} = 25.9 \text{ km s}^{-1} \text{ kpc}^{-1}$ and differs marginally from the Feast & Whitelock (1997) analysis of Hipparcos Cepheids of $27.19 \pm 0.87 \text{ km s}^{-1} \text{ kpc}^{-1}$. Recent studies, using samples of OB-type stars within 3 kpc of the Sun (excluding Gould’s Belt stars and based on Hipparcos data augmented with photometrically determined distances), arrive at $A - B$ between $30 \text{ km s}^{-1} \text{ kpc}^{-1}$ (Uemura et al. 2000) and $32 \text{ km s}^{-1} \text{ kpc}^{-1}$ (Miyamoto & Zhu 1998; Elias, Alfaro & Cabrera-Caño 2006), with uncertainties of about $\pm 1.5 \text{ km s}^{-1}$.

Our value for Θ_0/R_0 is in excellent agreement with that determined directly from the *apparent* proper motion of Sgr A* (the supermassive black hole at the Galactic center) of $6.379 \pm 0.024 \text{ mas y}^{-1}$ (Reid & Brunthaler 2004). One expects a supermassive black hole to be stationary at the dynamical center of the Galaxy to better than $\sim 1 \text{ km s}^{-1}$ (Chatterjee, Hernquist & Loeb 2002; Dorband, Hemsendorf & Merritt 2003; Reid & Brunthaler 2004). Hence, Sgr A*’s *apparent* motion should be dominated by the effects of the Galactic orbit of the Sun. After correcting for the Solar Motion of 5.25 km s^{-1} in the direction of Galactic rotation (Dehnen & Binney 1998), Sgr A*’s apparent motion yields a global estimate of $\Theta_0/R_0 = 29.45 \pm 0.15 \text{ km s}^{-1} \text{ kpc}^{-1}$. Thus, there is excellent agreement between this and our global and direct methods for measuring Θ_0/R_0 .

Coupling the Sgr A* motion result of $\Theta_0/R_0 = 29.45 \pm 0.15 \text{ km s}^{-1} \text{ kpc}^{-1}$ with estimates of R_0 from stellar orbits in the Galactic center (Ghez et al. 2008; Gillessen et al. 2009) of $8.4 \pm 0.4 \text{ kpc}$ yields $\Theta_0 = 247 \pm 12 \text{ km s}^{-1}$. This result is also a direct and global measurement of Θ_0 and is independent of our result from parallaxes and proper motions of star forming regions. Combining the Galactic center and star forming region estimates gives $\Theta_0 = 250 \pm 10 \text{ km s}^{-1}$. This value for Θ_0 is consistent with values derived by applying the Tully-Fisher relation to the Milky Way and using the infrared luminosity of the Milky Way determined by the COBE satellite’s DIRBE instrument (Malhotra et al. 1996). It seems clear that Θ_0 is near the upper end of the range of estimates in the literature. We note that both the Galactic center stellar orbits and star forming region parallax results assume the Hipparcos solar motion of 5.25 km s^{-1} in the direction of Galactic rotation. Only if the interpretation of the asymmetrical drift is incorrect or if the entire solar neighborhood orbits the Galactic center $\sim 30 \text{ km s}^{-1}$ slower than the Galaxy spins could Θ_0 be equal to the IAU recommended value of 220 km s^{-1} .

We have determined the rotation speed of the Milky Way at the radius of the Sun to be $\approx 250 \text{ km s}^{-1}$ and the rotation curve to be nearly flat or slightly rising with distance from

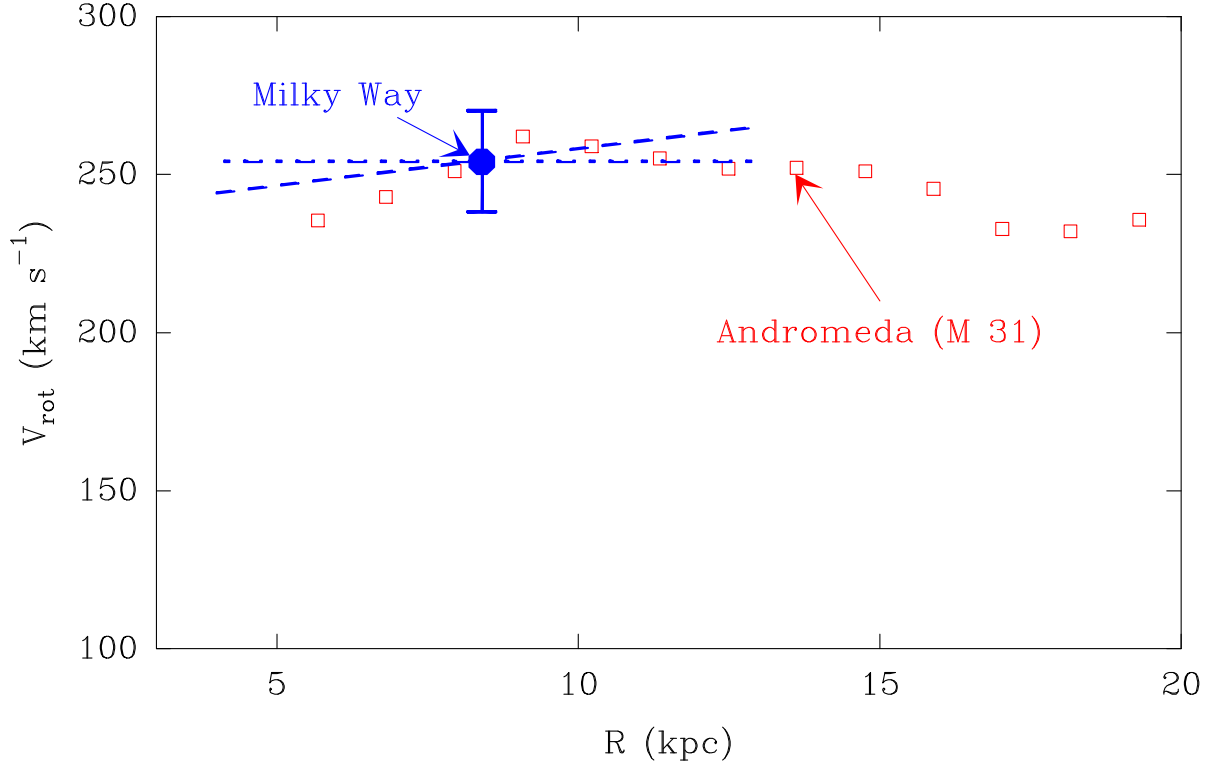


Fig. 6.— Rotation speed versus radius for the Andromeda galaxy and the Milky Way. The *red squares* are based on HI observations of Andromeda tabulated by Carignan et al. (2006). The *blue filled circle* is our best estimate of $\Theta_0 = 254 \pm 16 \text{ km s}^{-1}$ at $R_0 = 8.4 \text{ kpc}$ for the Milky Way, derived from the parallax and proper motions of high mass star forming regions. The *blue dot-dashed line* is for a flat rotation curve, and the *blue dashed line* corresponds to a slightly rising rotation curve of $2.3 \text{ km s}^{-1} \text{ kpc}^{-1}$ (see §3.3). These lines are plotted over the range of Galactocentric radii sampled by the parallax and proper motion results. Note that these two galaxies have nearly identical rotation speeds over this range.

the Galactic center. These values are nearly identical to those of the Andromeda galaxy (M31) as shown in Fig. 6. The rotation curve of Andromeda, determined from HI emission by Carignan et al. (2006) based on interferometric observations of Unwin (1983), indicates a speed of 251 km s^{-1} at a radius of 8 kpc, a slightly rising curve out to about 15 kpc, and a slow dropoff to about 225 km s^{-1} beyond 20 kpc. The most straightforward interpretation of the similarities of the rotation curves for the Milky Way and Andromeda is that these two galaxies are nearly equal in size and mass.

Finally, we note that Reid & Brunthaler (2004) placed a strong upper limit of $-0.4 \pm 0.9 \text{ km s}^{-1}$ for the component of peculiar motion of Sgr A* *perpendicular* to the plane of the Galaxy. However, the determination of the component in the direction of Galactic rotation was considerably less accurate: $18 \pm 7 \text{ km s}^{-1}$, as one must remove the uncertain effects of the solar orbit. Reid & Brunthaler did this by removing $27.19 \pm 0.87 \text{ km s}^{-1} \text{ kpc}^{-1}$, based on Hipparcos measurements of Oort’s constants ($A - B$) by Feast & Whitelock (1997), from the observed motion of Sgr A* in the Galactic plane of $29.45 \text{ km s}^{-1} \text{ kpc}^{-1}$. This method assumes that $\Theta_0/R_0 = A - B$ and that estimates of the shear and vorticity of nearby stars from Hipparcos data indicate the large-scale differential rotation of the Galaxy and are not subject to local irregularities in the solar neighborhood. Since we now have a direct, global estimate of $\Theta_0/R_0 = 30.3 \pm 0.9 \text{ km s}^{-1} \text{ kpc}^{-1}$, we find the peculiar motion of Sgr A* in the direction of Galactic rotation to be $-7.2 \pm 8.5 \text{ km s}^{-1}$, with little sensitivity to R_0 (adopted to be 8.5 kpc here). This adds additional strong evidence that Sgr A* is a supermassive black hole, which is nearly stationary at the dynamical center of the Galaxy.

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7. Appendix

Since we have measured the distance, LSR velocity and proper motion of each source, we know its full 3-dimensional location in the Galaxy and full space motion. Given a model of Galactic rotation, we can then calculate the non-circular (peculiar) velocity of each source. While this calculation is conceptually simple, in practice, there are some subtleties and sign convention issues that can lead to errors, and so here we present the necessary formulae (and FORTRAN source code in on-line material).

The required Galactic and Solar Motion parameters are given in Table 6, and those associated with the source are defined in Table 7. A schematic depiction of these parameters is given in Fig. 7. We assume that the Sun is in the Galactic plane and calculate a source’s peculiar motion (i.e. with respect to a circular Galactic orbit) as follows:

We convert v_{LSR} to a heliocentric frame, v_{Helio} , by adding back the component of the Standard Solar Motion in the radial direction that had been removed from the observed Doppler shift to calculate v_{LSR} . Note that one needs to use the (old) Standard Solar Motion, which defines the LSR frame, and *not* the best values available today. Generally, observatories have adopted a value of 20 km s^{-1} toward $\alpha(1900) = 18^{\text{h}}, \delta(1900) = +30^{\text{d}}$ for the Standard Solar Motion. Precessing these coordinates to the epoch of observation (≈ 2006) and converting to Galactic Cartesian coordinates yields the $(U_{\odot}^{\text{Std}}, V_{\odot}^{\text{Std}}, W_{\odot}^{\text{Std}})$ values listed in Table 6. Then,

$$v_{\text{Helio}} = v_{\text{LSR}} - (U_{\odot}^{\text{Std}} \cos \ell + V_{\odot}^{\text{Std}} \sin \ell) \cos b - W_{\odot}^{\text{Std}} \sin b \quad .$$

Rotate the equatorial heliocentric frame $(\mu_{\alpha}, \mu_{\delta}, v_{\text{Helio}})$ to a Galactic heliocentric frame $(\mu_l, \mu_b, v_{\text{Helio}})$. This is a rotation about a radial axis and is defined by the IAU in B1950.0 coordinates (Blaauw et al. 1960). For coordinates in J2000, Reid & Brunthaler (2004)

Table 6. Galactic and Solar Parameters and Nominal Values

Parameter	Value	Definition
R_0	8.5 kpc	Distance to the GC (IAU value)
Θ_0	220 km s ⁻¹	Rotation speed of LSR (IAU value)
Θ_s	220 km s ⁻¹	Rotation speed of Galaxy at source
U_{\odot}^{Std}	10.3 km s ⁻¹	Standard Solar Motion toward GC
V_{\odot}^{Std}	15.3 km s ⁻¹	Standard Solar Motion toward $\ell = 90^\circ$
W_{\odot}^{Std}	7.7 km s ⁻¹	Standard Solar Motion toward NGP
U_{\odot}^H	10.0 km s ⁻¹	Hipparcos Solar Motion toward GC
V_{\odot}^H	5.2 km s ⁻¹	Hipparcos Solar Motion toward $\ell = 90^\circ$
W_{\odot}^H	7.2 km s ⁻¹	Hipparcos Solar Motion toward NGP

Note. — GC: the Galactic Center; LSR: Local Standard of Rest; NGP: North Galactic Pole. The Standard Solar Motion must be used to convert from v_{LSR} to v_{Helio} , since (hopefully) all observatories have used this definition. The values given above come from an assumed Solar Motion of 20 km s⁻¹ toward R.A.(1900)=18^h and Dec.(1900)-30° precessed to J2000.0. Hipparcos Solar Motion values are from Dehnen & Binney (1998).

Table 7. Source Parameter Definitions

Parameter	Definition
ℓ	Galactic longitude
D	Distance from Sun ($1/\pi_s$)
D_p	Distance from Sun projected in plane
R_p	Distance from GC projected in plane
v_{LSR}	LSR radial velocity
v_{Helioc} ...	Heliocentric radial velocity
μ_α	Proper motion toward East
μ_δ	Proper motion toward North
β	Angle: Sun–GC–source
U_s	Peculiar motion locally toward GC
V_s	Peculiar motion locally in direction of Galactic rotation
W_s	Peculiar motion toward NGP

Note. — GC: the Galactic Center; LSR: Local Standard of Rest; NGP: North Galactic Pole.

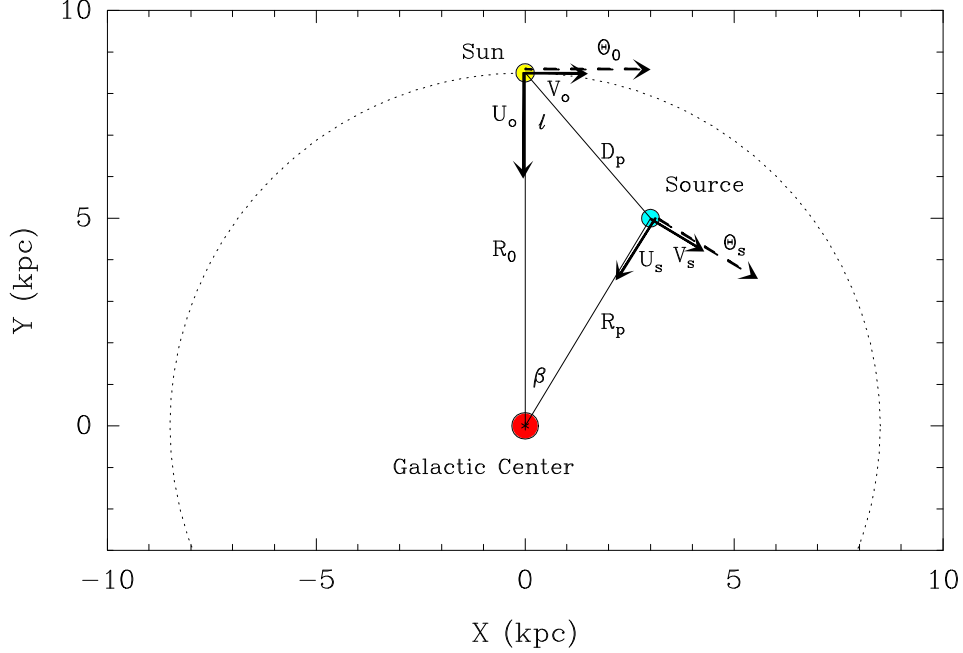


Fig. 7.— Schematic depiction of source and Galactic parameters.

give the right ascension and declination of the NGP as $\alpha_P = 12^h 51^m 26.2817^s$ and $\delta_P = 27^\circ 07' 42.013''$, respectively, and the zero of longitude is the great semicircle originating at the NGP at the position angle $\theta = 122^\circ.932$. Galactic latitude can be obtained from

$$\sin b = \sin \delta \cos (90^\circ - \delta_P) - \cos \delta \sin (\alpha - \alpha_P - 6^h) \sin (90^\circ - \delta_P) \quad .$$

A useful angle ϕ can be determined (between 0° and 360°) from

$$\sin \phi = \left(\cos \delta \sin (\alpha - \alpha_P - 6^h) \cos (90^\circ - \delta_P) + \sin \delta \sin (90^\circ - \delta_P) \right) / \cos b$$

and

$$\cos \phi = \cos \delta \cos (\alpha - \alpha_P - 6^h) / \cos b \quad ,$$

and then Galactic longitude follows from

$$\ell = \phi + (\theta - 90^\circ) \quad .$$

Proper motions in Galactic coordinates (μ_l, μ_b) can be easily calculated from motions in equatorial coordinates (μ_α, μ_δ) by differencing (ℓ, b) values for coordinates determined, say, one year apart. This usually requires 64-bit precision in the calculations. Note that μ_l

will naturally be defined positive in the direction of increasing Galactic longitude, which is *counter* to Galactic rotation.

Convert the proper motions to linear speeds (by multiplying by distance) via

$$v_\ell = D\mu_l \cos b \quad \text{and} \quad v_b = D\mu_b \quad ,$$

where $\mu_l \cos b$ is the actual motion tangent to the sky in the direction of Galactic longitude.

We now convert from spherical to Cartesian Galactic coordinates at the location of the Sun.

$$\begin{aligned} U_1 &= (v_{Helio} \cos b - v_b \sin b) \cos \ell - v_\ell \sin \ell \quad , \\ V_1 &= (v_{Helio} \cos b - v_b \sin b) \sin \ell + v_\ell \cos \ell \quad , \\ W_1 &= v_b \cos b + v_{Helio} \sin b \quad . \end{aligned}$$

Next add the full orbital motion of the Sun, using the best values of the solar motion and the circular rotation of the Galaxy at the Sun ($U_\odot^H, V_\odot^H + \Theta_0, W_\odot^H$).

$$U_2 = U_1 + U_\odot^H \quad , \quad V_2 = V_1 + V_\odot^H + \Theta_0 \quad , \quad W_2 = W_1 + W_\odot^H \quad .$$

The Galactocentric distance to the source projected onto the Galactic plane is given by

$$R_p^2 = R_0^2 + D_p^2 - 2R_0 D_p \cos \ell \quad ,$$

where $D_p = D \cos b$. The angle β between the Sun and the source as viewed from the GC can be determined in all cases (i.e. from 0° to 360°) from

$$\sin \beta = \frac{D_p}{R_p} \sin \ell \quad \text{and} \quad \cos \beta = \frac{R_0 - D_p \cos \ell}{R_p} \quad ,$$

Rotate the vector (U_2, V_2, W_2) through the angle β in the plane of the Galaxy and remove circular Galactic rotation at the source to yield (U_s, V_s, W_s) :

$$\begin{aligned} U_s &= U_2 \cos \beta - V_2 \sin \beta \quad , \\ V_s &= V_2 \cos \beta + U_2 \sin \beta - \Theta_s \quad , \\ W_s &= W_2 \quad . \end{aligned}$$

The vector (U_s, V_s, W_s) gives the non-circular (peculiar) motion of the source in a Cartesian Galactocentric frame, where U_s is radially inward toward the Galactic center (as viewed by the source), V_s is in the local direction of Galactic rotation and W_s is toward the north Galactic pole.