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Extraction of relative magnification matrices from VLBI observations of gravitational lens systems

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Abstract. To get more reliable constraints on the reconstruction of gravitational lenses, we are working on a method to extract relative magnification matrices of the lensed images directly from VLBI visibilities. Instead of independently reconstructing multiple images of the source, we describe the sky brightness distribution by that of one of the images, plus a set of linear coordinate transformations for the other images characterising the action of the lens; parameters then are to be adjusted by fitting visibilities.
To parameterise the brightness profile of the source we use the shapelet formalism as introduced by Refregier (2001). Within this formalism a localised object is linearly decomposed with respect to a complete set of orthonormal basis functions. These basis functions have a number of remarkable mathematical properties – simple behaviour under Fourier transform, convolution and coordinate transformations. This makes them a promising candidate to model compact sources as observed in strong gravitational lensing.

1. Introduction

VLBI serves as a powerful tool for high resolution studies of gravitational lens systems. Identification of substructure in the multiple images of a distant source allows computation of relative magnification matrices, which are used as constraints on the reconstruction of the lens.

While past investigations (e.g. Barkana et al. 1999) of the gravitational lens mapping depended on the analysis of reconstructed images, our aim is to perform the same type of analysis directly on the measured visibilities. This requires an efficient method to model the sky brightness distribution and analytical access to the related visibilities.

These requirements are met by the shapelet formalism as introduced by Refregier (2001).

2. Shapelets

The solutions to the quantum harmonic oscillator (QHO)

$$\phi_n(x) = \frac{1}{\sqrt{2^n \sqrt{\pi}\, n!}} H_n(x)\, e^{-x^2/2}\,, \tag{1}$$

where n is an integer and $H_n(x)$ is a Hermite polynomial of order n, form a complete set of orthonormal functions, that can be used as set of basis functions in function space (e.g. Elbaz 1998). The $\phi_n(x)$ can be thought of as perturbations around the Gaussian $\phi_0(x)$ and now will be referred to as *Shapelets* (Refregier 2001).

2.1. Basis functions

To describe a 1-D object of characteristic scale β from (1) one defines dimensional basis functions

$$B_n(x;\beta) = \frac{1}{\sqrt{\beta}}\, \phi_n(\beta^{-1}x)\,, \tag{2}$$

where the scale parameter β is typically chosen to be close to the size of the object. The functions $B_n(x;\beta)$ again are complete and orthonormal, such that an object profile $f(x)$ can be expanded as

$$f(x) = \sum_{n=0}^{\infty} f_n\, B_n(x;\beta)\,. \tag{3}$$

From the orthonormality relation for the $B_n(x;\beta)$, the shapelet coefficients are given by

$$f_n = \int_{-\infty}^{+\infty} dx\, f(x)\, B_n(x;\beta)\,. \tag{4}$$

The basis functions for 2-dimensional objects can now be constructed by taking the tensor product of two 1-dimensional basis functions:

$$\phi_{\boldsymbol{n}}(\boldsymbol{x}) = \phi_{n_1}(x_1)\, \phi_{n_2}(x_2)\,, \tag{5}$$

$$B_{\boldsymbol{n}}(\boldsymbol{x};\beta) = \frac{1}{\beta}\, \phi_{\boldsymbol{n}}(\beta^{-1}\boldsymbol{x})\,. \tag{6}$$

where $\boldsymbol{x} = (x_1, x_2)$ and $\boldsymbol{n} = (n_1, n_2)$. These basis functions again form an orthonormal basis for smooth, integrable functions in two variables, such that the 2-D profile $f(\boldsymbol{x})$ of an object can be decomposed as

$$f(\boldsymbol{x}) = \sum_{n_1,n_2=0}^{\infty} f_{\boldsymbol{n}}\, B_{\boldsymbol{n}}(\boldsymbol{x};\beta)\,. \tag{7}$$

2.2. Transformation properties

The shapelet basis functions have a number of remarkable mathematical properties that can be used for photometric purposes and the analysis of interferometric data (Chang & Refregier 2002):

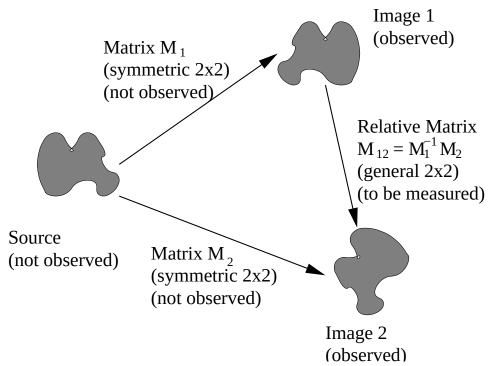

Fig. 1. Multiple imaging of a distant object in gravitational lensing.

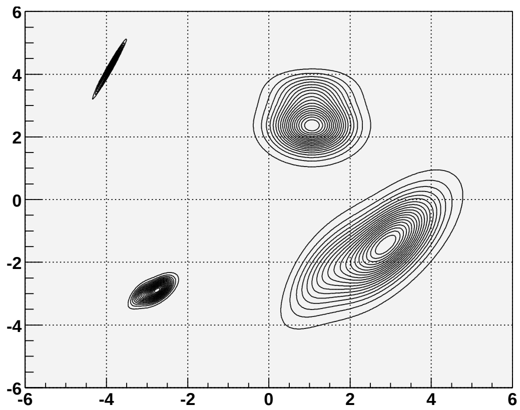

Fig. 2. Brightness distribution generated using a shapelet-based source model and a set of linear coordinate transformations.

– The Fourier transform of the dimensional basis function $B_n(x;\beta)$ is given by

$$\widetilde{B}_n(k;\beta) = i^n\, B_n(k;\beta^{-1})\,, \tag{8}$$

that is a basis function with inverted scale $\beta \to \beta^{-1}$.

– The total flux

$$F \equiv \int d^2x\, f(\boldsymbol{x}) \tag{9}$$

of an object can be written completely in terms of the coefficients $f_{n_1 n_2}$.

– For a general linear coordinate transformation of the form

$$\boldsymbol{x} \;\rightarrow\; \boldsymbol{x}' = (1+\mathbf{M})\boldsymbol{x} + \boldsymbol{\epsilon}\,, \tag{10}$$

where $\mathbf{M}$ is a 2×2 matrix and $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2)$ is a small displacement, the transformed object profile $f'(\boldsymbol{x}') = f(\boldsymbol{x}(\boldsymbol{x}'))$ can be obtained from $f(\boldsymbol{x})$, using the operator formalism established from quantum theory.

– Convolution (e.g. under action of PSF) of functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$

$$h(\boldsymbol{x}) \;=\; (f * g)(\boldsymbol{x})$$

can be expressed in terms of shapelet coefficients

$$h_{\boldsymbol{n}} \;=\; \sum_{\boldsymbol{m},\boldsymbol{l}} C_{\boldsymbol{n},\boldsymbol{m},\boldsymbol{l}} f_{\boldsymbol{m}} g_{\boldsymbol{l}}\,, \tag{11}$$

where $C_{\boldsymbol{n},\boldsymbol{m},\boldsymbol{l}}$ is the 2-dimensional convolution tensor.

3. Description of gravitational lens systems forming multiple images

Linearisation of the lens equation yields that the lens mapping can be described locally using symmetric 2×2 matrices. However it turns out (Fig. 1), that these matrices cannot be accessed by observation – only relative matrices can be observed (e.g. Narayan & Bartelmann 1996).

Considering a general linear coordinate transformation, a function $f(\boldsymbol{x})$ transforms as

$$f(\boldsymbol{x}) \;\rightarrow\; f'(\boldsymbol{x}') = f(\boldsymbol{x}(\boldsymbol{x}')) = f(\mathbf{M}^{-1}(\boldsymbol{x}' - \boldsymbol{s}))\,, \tag{12}$$

where $\mathbf{M}$ is a general 2×2 matrix and $\boldsymbol{s}$ a vector to account for a shift in position. Using one such linear coordinate transformation for each of the lensed images m we can write for the sky brightness distribution (Fig. 2):

$$\begin{aligned} I'(\boldsymbol{x}') \;&=\; \sum_m I'^{(m)}(\boldsymbol{x}') \\ &=\; \sum_m I\Big(\mathbf{M}^{(m)-1}(\boldsymbol{x}' - \boldsymbol{s}^{(m)})\Big) \\ &=\; \sum_{m,n} f_n\, B_n\Big(\mathbf{M}^{(m)-1}(\boldsymbol{x}' - \boldsymbol{s}^{(m)});\beta\Big) \end{aligned} \tag{13}$$

Due to the properties of the shapelet basis functions under Fourier transform, eq. (8), the Fourier transform of the sky brightness distribution (13), $\widetilde{I}(u,v)$, again can be written completely in terms of the shapelet basis functions.

With this we are working on a χ^2 fit

$$\chi^2 \;=\; \sum_{u,v} \left[\frac{V_{\mathrm{m}}(u,v) - \widetilde{I}(u,v)}{\sigma(u,v)}\right]^2\,, \tag{14}$$

to minimise the difference between measured visibilities V_{m} and model visibilities $\widetilde{I}$, as given by the shapelet coefficients $f_{\boldsymbol{n}}$ and the coordinate transformations parameters.

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