Principles of Interferometry

Hans-Rainer Klöckner

IMPRS Black Board Lectures 2014

acknowledgement

- Mike Garrett lectures
- Uli Klein lectures
- Adam Deller NRAO Summer School lectures
- WIKI – for technical stuff

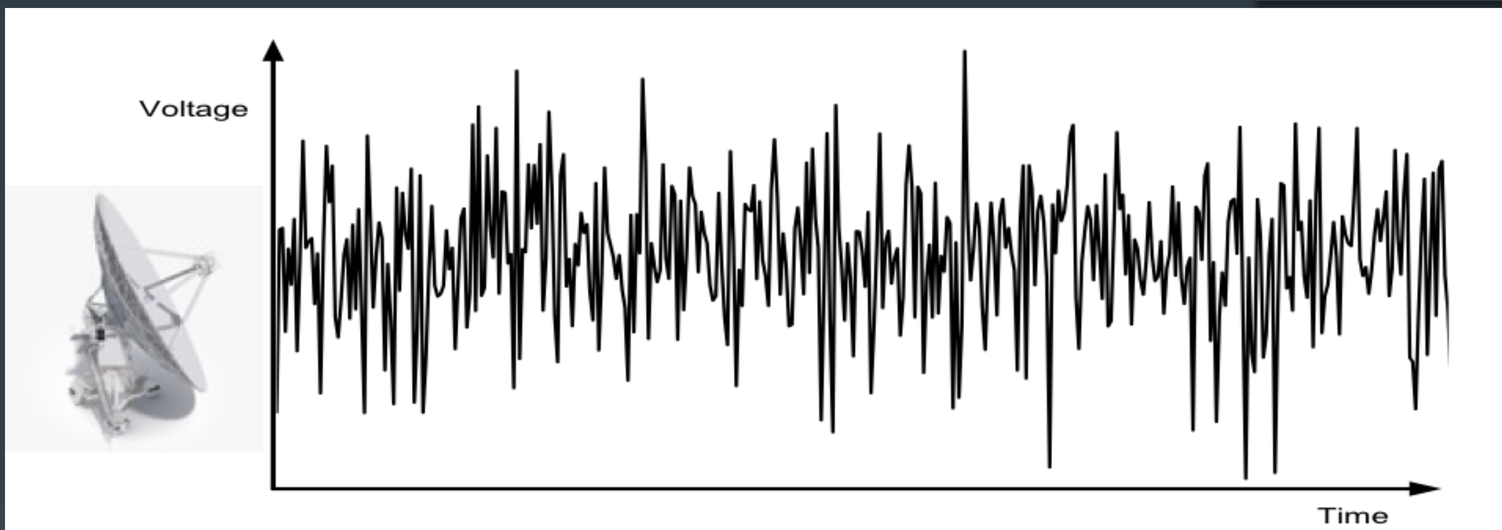
Lecture 3

- radio astronomical system
- heterodyne receivers
- low-noise amplifiers
- system noise performance
- data sampling/representation
- Fourier transformation

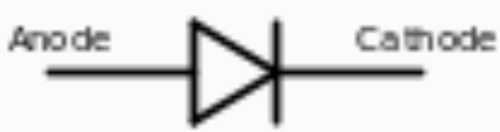
a basic system

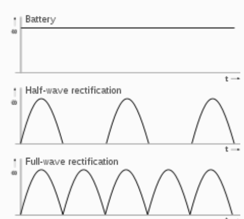
relate the voltages measured at the receiver system to the antenna temperature

$$S_\nu = \frac{2k}{A_{eff}} T_A$$



alternating current (AC)





direct current (DC)

detector input power ~ 10^{-5} W

$$P = k \cdot \Delta\nu \cdot T_{\rm sys}$$

Tsys = 20 K, $\Delta\nu$ 50 MHz

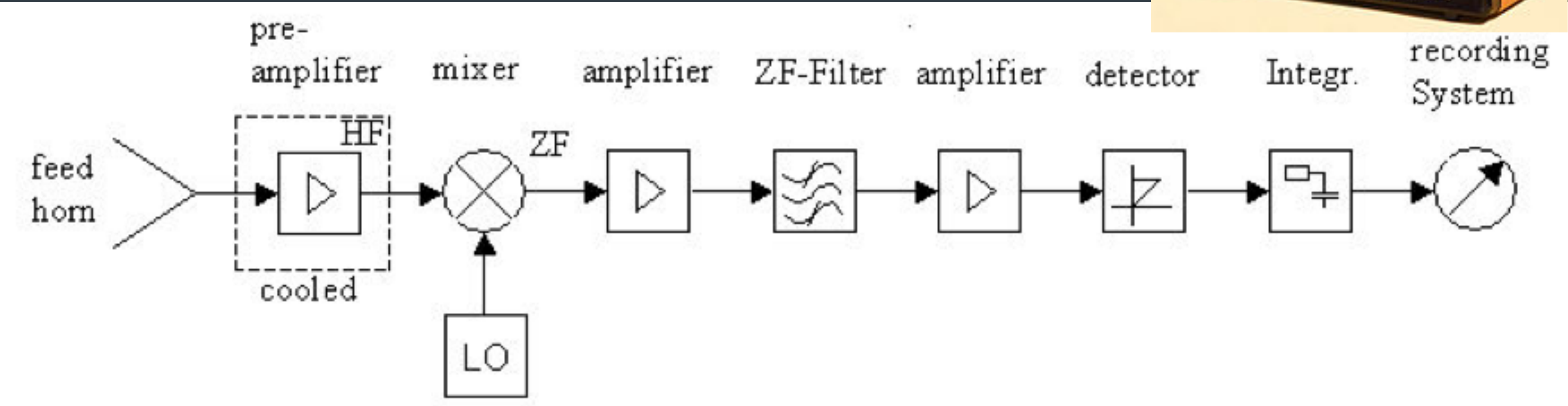
P = 1.4 10^{-14} W

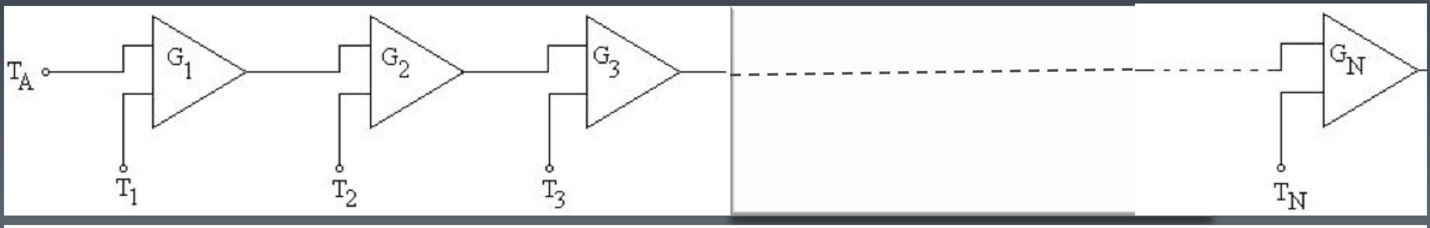
~10^{8} amplification / gain

heterodyne receiver

after all its just listening to radio

the most used setup

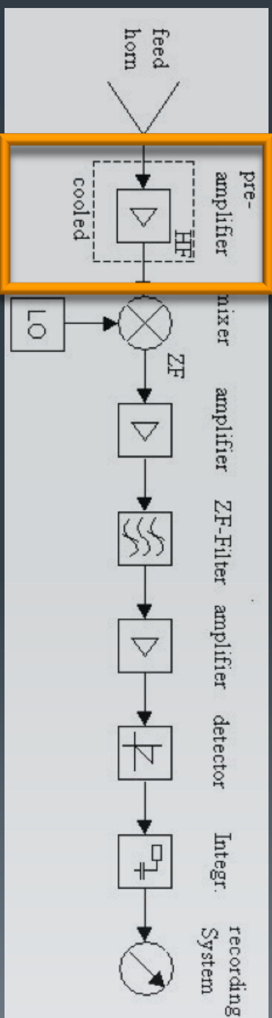


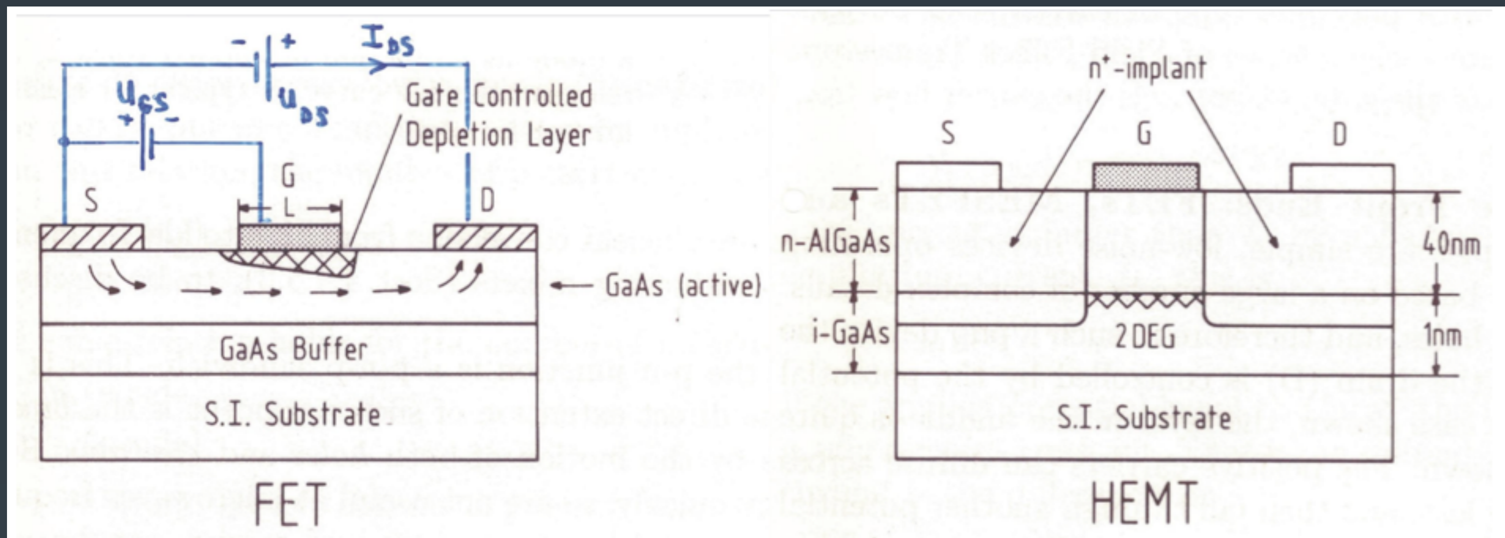


$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \ldots + \frac{T_N}{G_1 \cdot G_2 \cdots G_{N-1}}$$

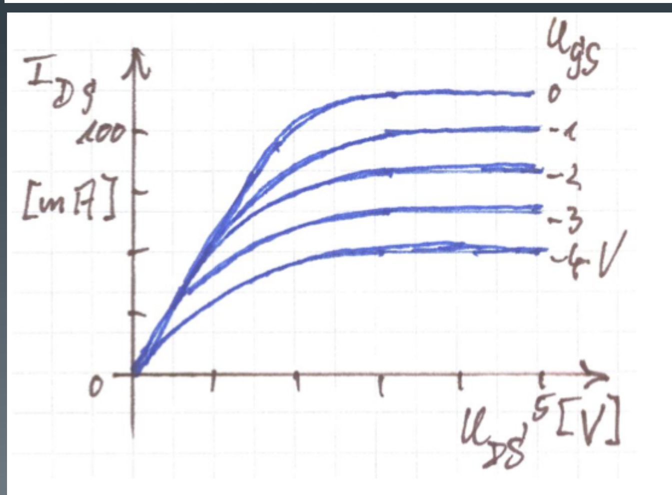
T_1 needs cooling

low noise amplifier



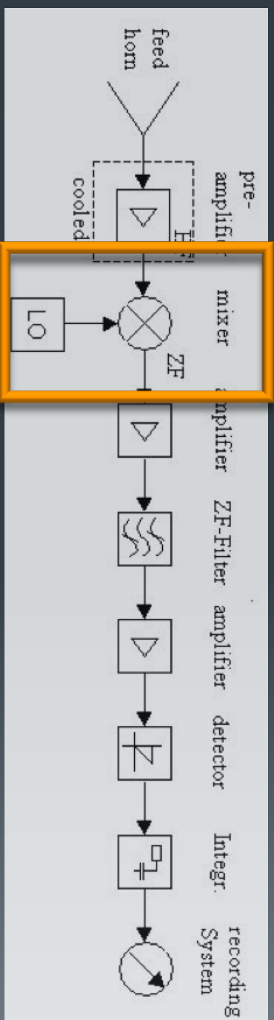


High Electron Mobility Transistor

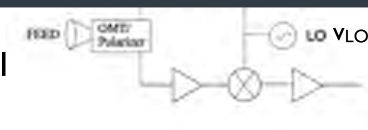


need to stay in the linear regime

mixer – frequency down conversion



A typical receiver tries to down-convert the "sky signal" or "Radio Frequency" (or RF) to a lower, "Intermediate Frequency" (or IF) signal



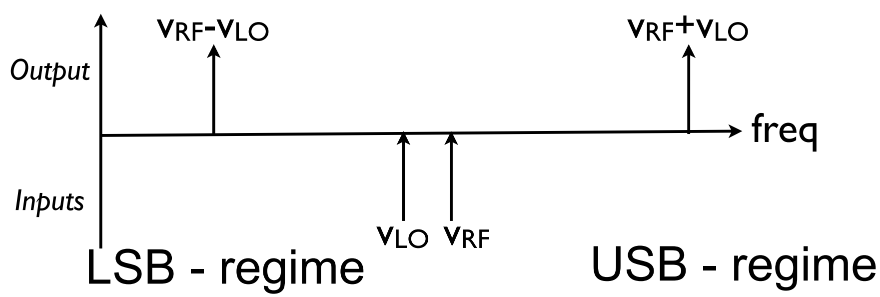
The reasons for doing this include: (i) signal losses (e.g. in cables) typically go as frequency^2; (ii) it is much easier to mainpulate the signal (e.g. amplify, filter, delay, sample/process/digitise it) at lower frequencies.

We use so-called "heterodyne" systems to mix the RF signal with a pure, monochromatic frequency tone, known as a Local Oscillator (or LO).

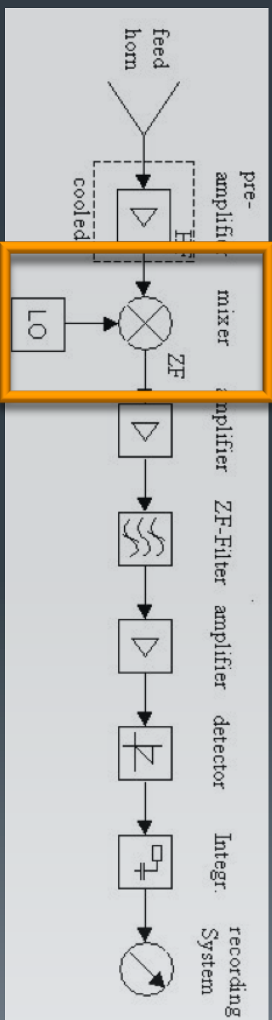
Consider an RF signal in a band centred on frequency $\nu_{RF,}$ and an LO with frequency ν_{LO}, these can be represented as two sine waves with angular frequencies w and w_o:

$$\nu_{IF} = \nu_{RF}\nu_{LO} \sim sin(\omega t)sin(\omega_o t) = 1/2(cos(\omega - \omega_o)t + cos(\omega + \omega_o)t)$$

– Difference frequency –   – Sum frequency –



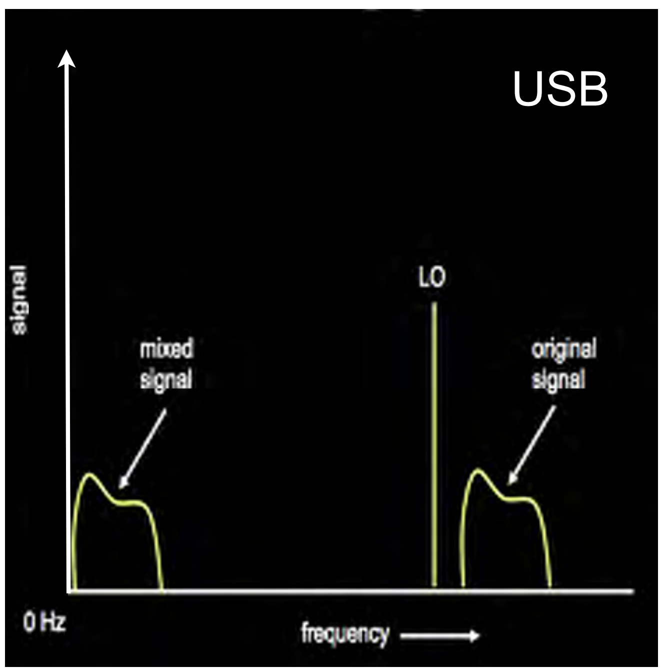
mixer – frequency down conversion



The higher frequency component ("sum frequency" $\nu_{RF}+\nu_{LO}$) is usually removed by a filter that is included in the LO electronics. Hence the process of down-conversion, takes a band with centre frequency ν_{RF} and converts it to a lower (difference) frequency, $\nu_{RF}-\nu_{LO}$.

USB = upper side band

LSB = lower side band

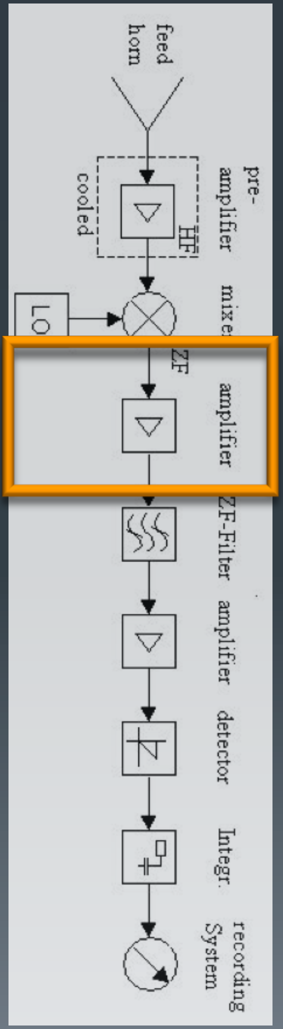


The mixer signal products preserves the noise characteristics of the input RF (sky) signal, but they contain an arbitrary phase-shift due to the unknown phase of the LO.

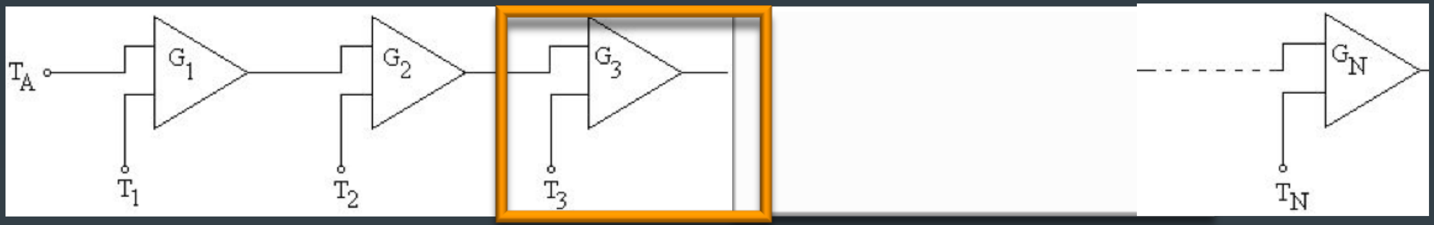
Usually there will be several mixers and frequency conversions in a receiver system. Eventually one edge of the frequency band reaches 0 Hz, known as a "base-band" or "video" signal.

At high frequencies (e.g. millimetre wavelengths), down-conversion occurs before amplification.

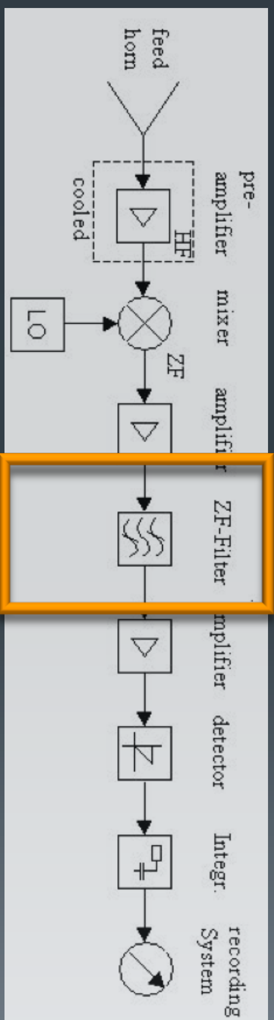
low noise amplifier

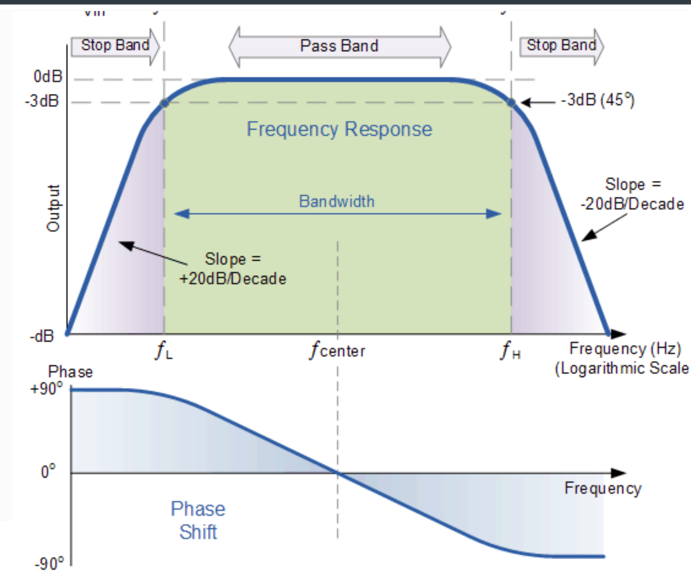


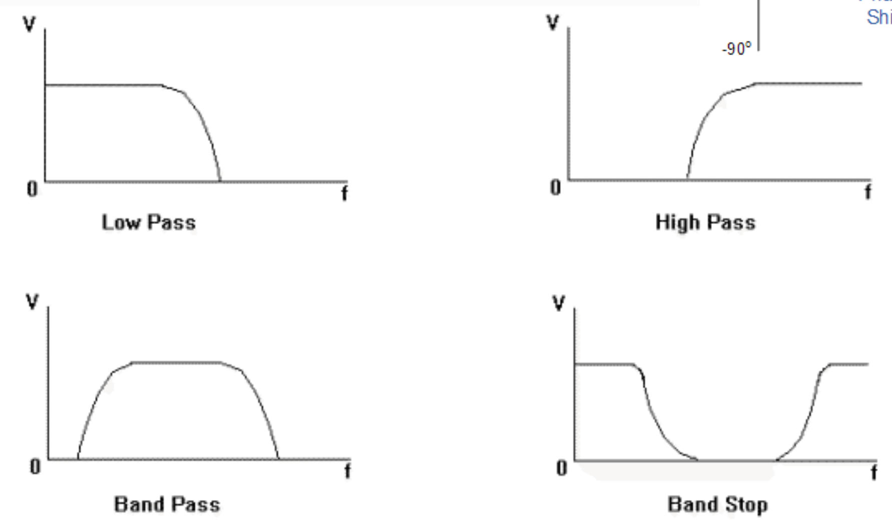
we have covered that already



bandpass filter

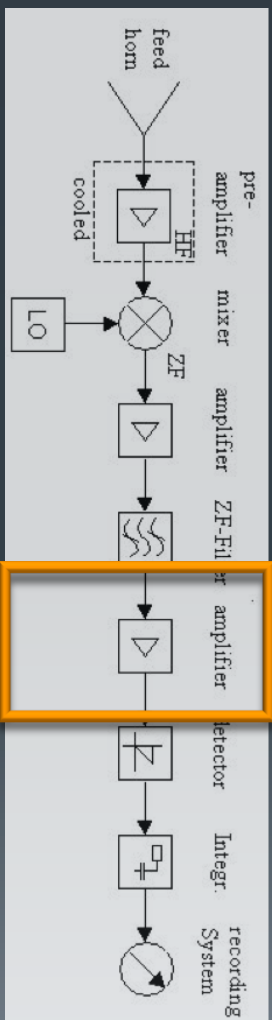


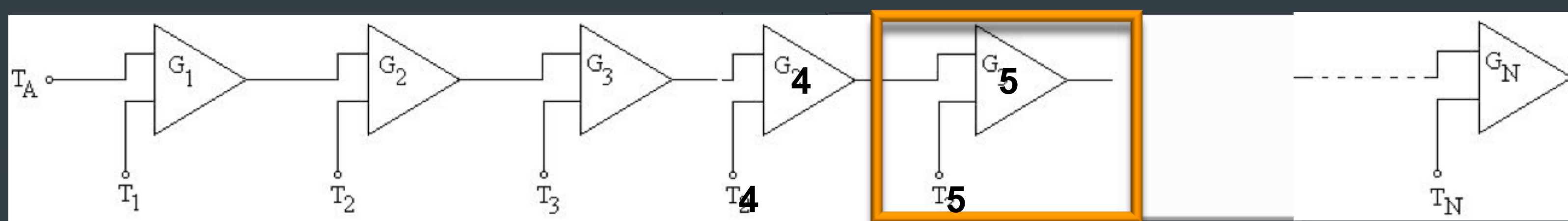




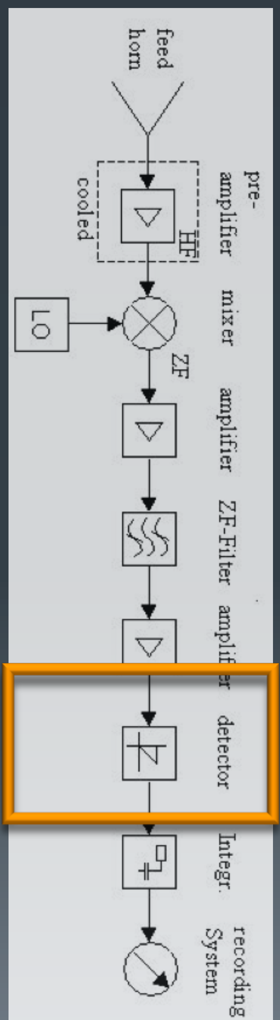
low noise amplifier

we have covered that already



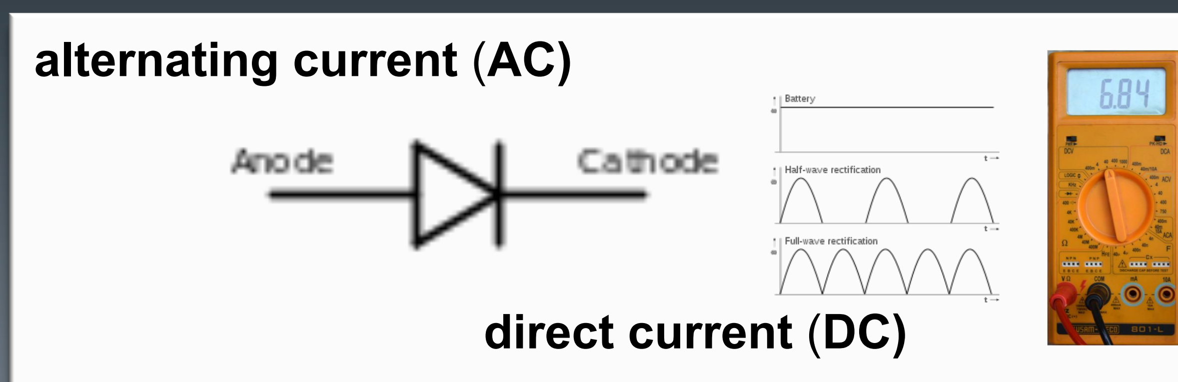


detector

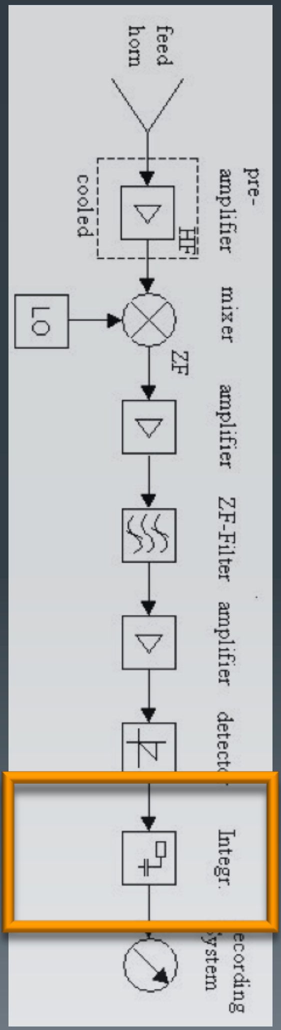


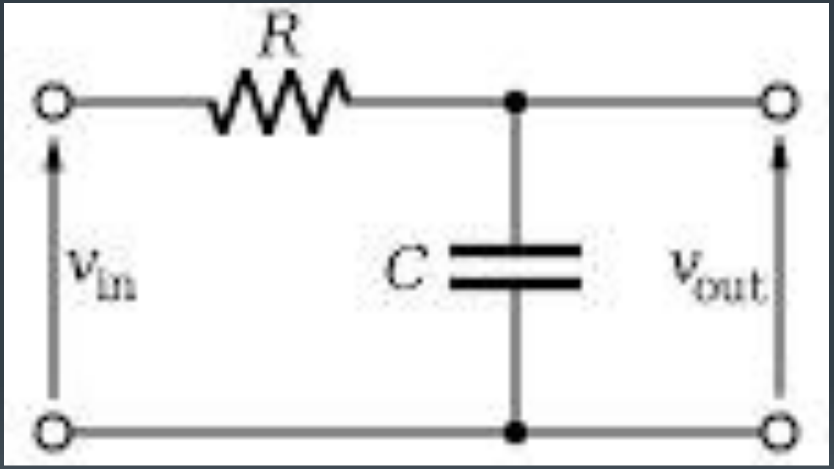
Since radio astronomy signals have the characteristics of white noise, the voltage induced in the receiver output alternates positively and negatively about zero volts. Any measurement of the Voltage expectation value or time average will read zero (e.g. hooking up a receiver to a DC voltmeter will not measure any signal).

What is needed is a non-linear device ($V_{out} = AV_{in}^2$) that will only measure the passage of the signal in one preferred direction (either positive or negative) i.e. we must incorporate a semiconductor diode into our measuring system



integrator



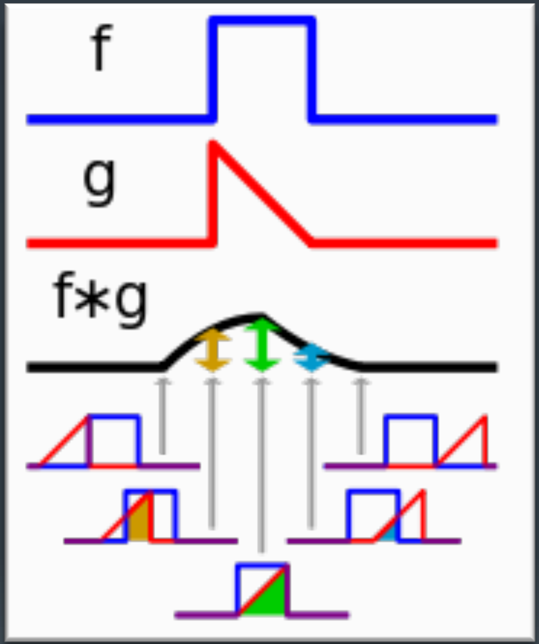


capacity needs time τ to charge

reads out the capacity

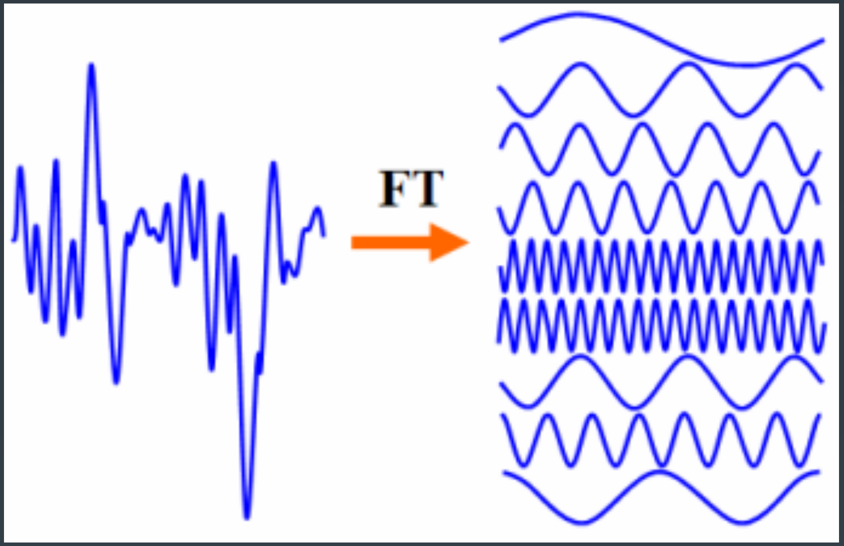
signal processing tools 1d

Convolution



$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

Fourier Transformation



$$F_f(t) = \int f(\nu) e^{2\pi i \nu t} d\nu$$

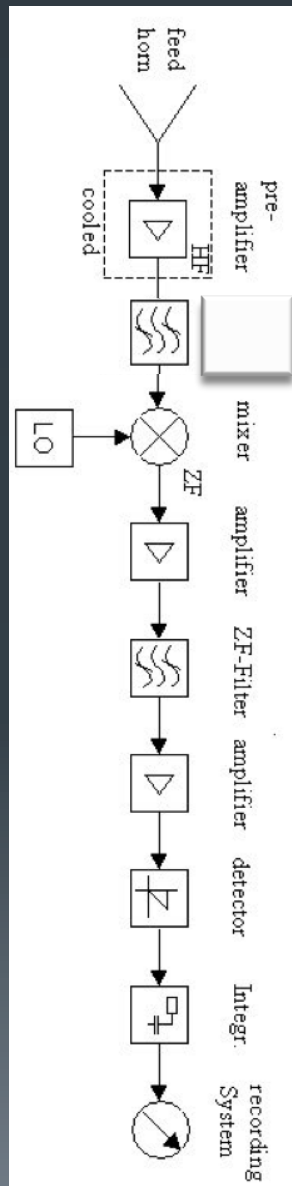
$$F_f(\nu) = \int f(t) e^{-2\pi i \nu t} dt$$

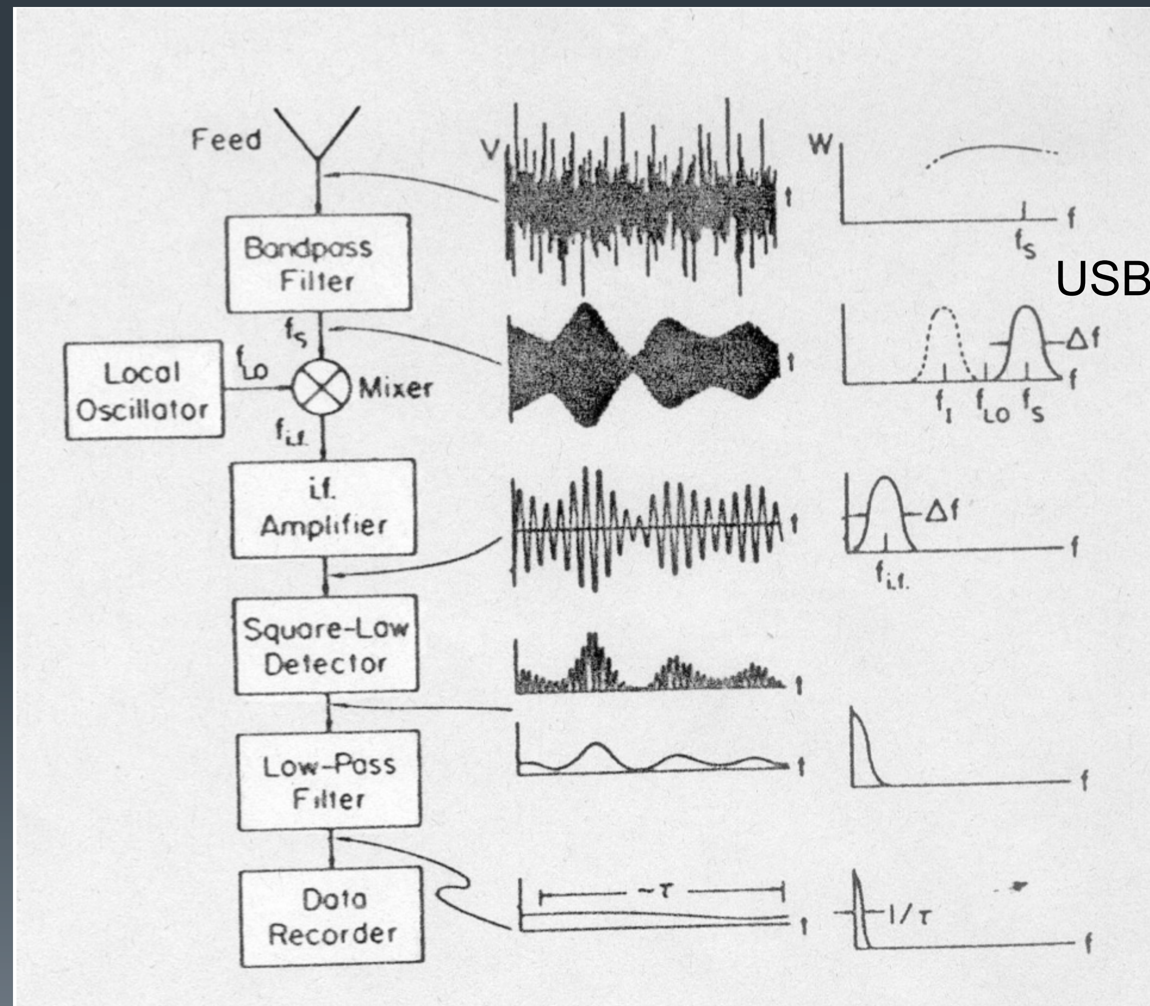
Convolution theorem

F(f * g) = F(f) F(g)

heterodyne receiver

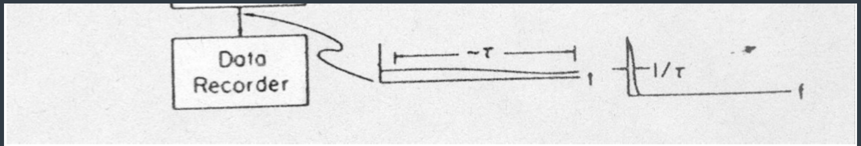
Fourier transformation
convolution theorem in action





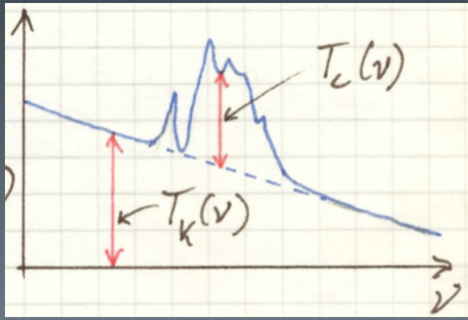
broad band IF signal

time dependent voltage U(τ)

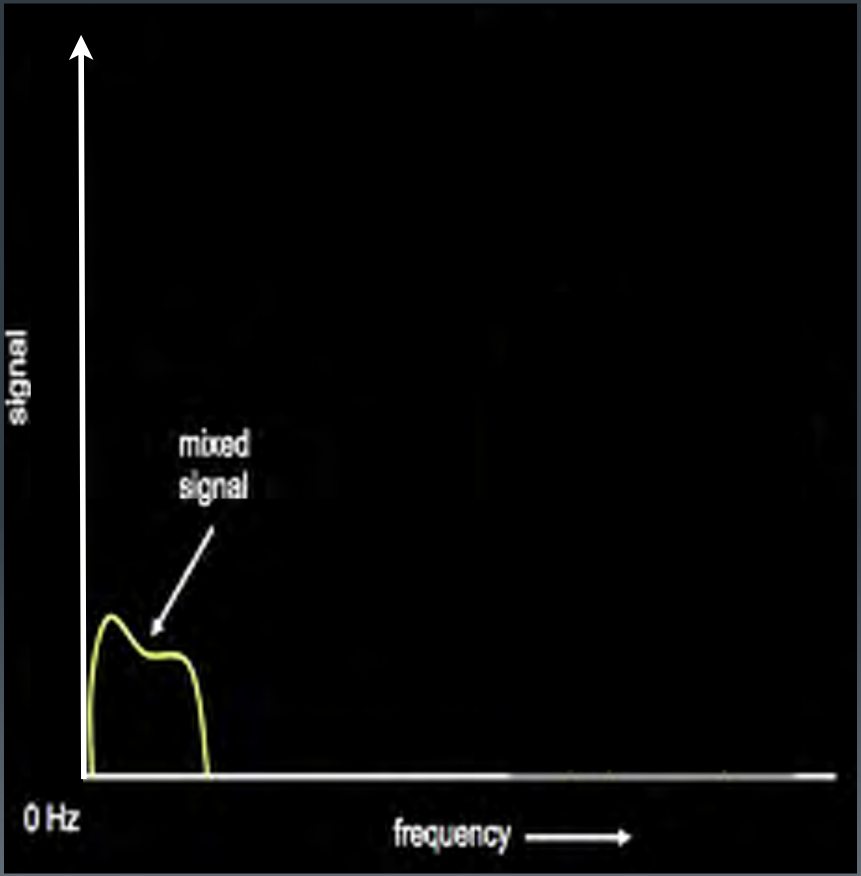


Continuum measurement
Power(τ) ~ |U²(τ)|

Line measurement
Power(ν) ~ |U²(ν)|



frequency dependent voltage U(ν)



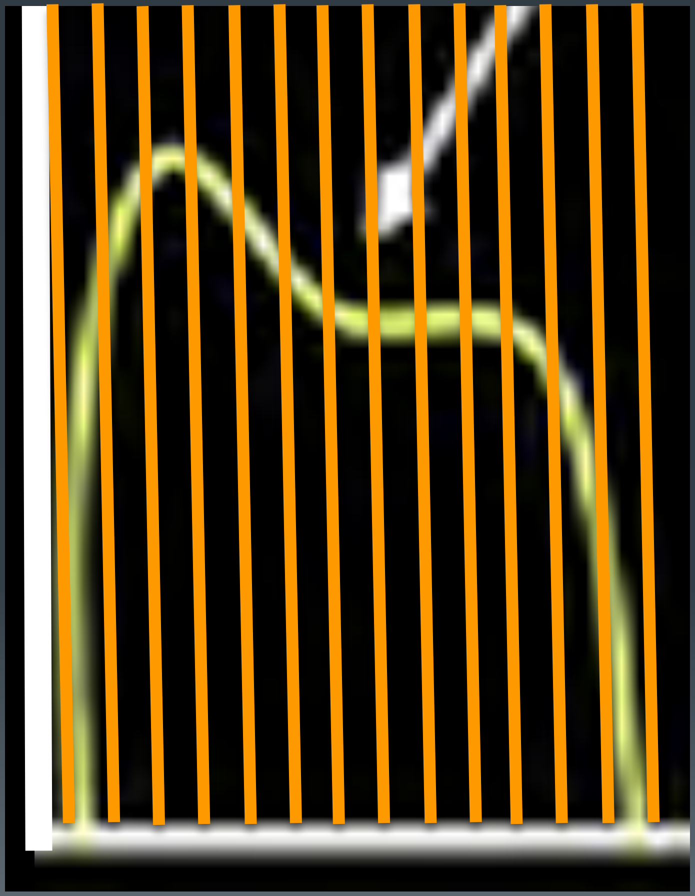
how to get P(ν) = |U²(ν)|
- approach 1 -

Theoretically

$$\tilde{U}(\nu) = \int_{-\infty}^{+\infty} U(t)\, e^{-i\, 2\pi\, \nu\, t}\, dt$$

$$P(\nu) = |\tilde{U}(\nu)|^2$$

Hardware



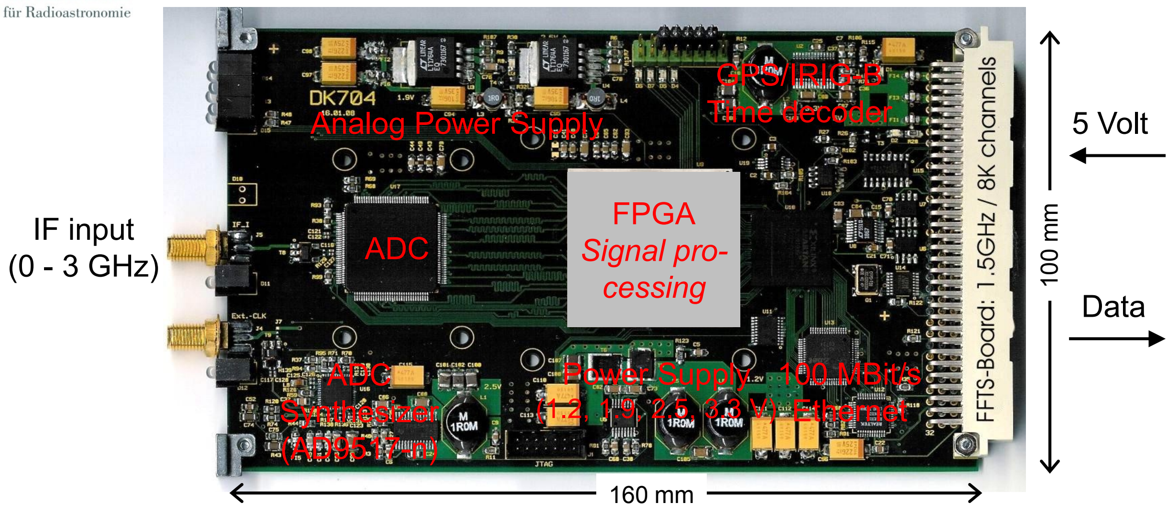
filters split signal into channels
feed each channel into detector

FFT spectrometer

Max-Planck-Institut für Radioastronomie

DIGITALLABOR

FFTS :: *1.5 GHz bandwidth Board*



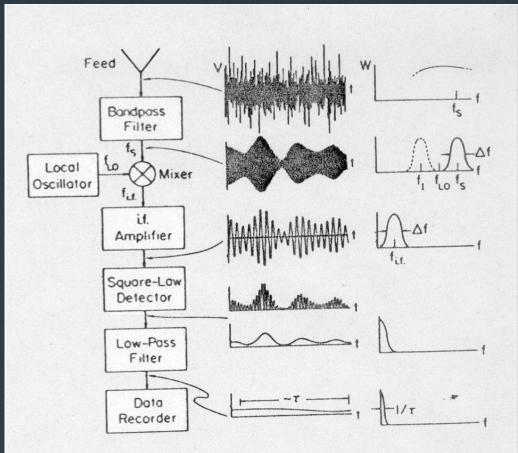
- Instantaneous bandwidth: 0.1 – 1.8 GHz
- Spectral resolution @ 1.5 GHz: 212 kHz
- Calibration- and aging free digital processing

01000110 01000110 01010100 01010011 – 01000010 01001011

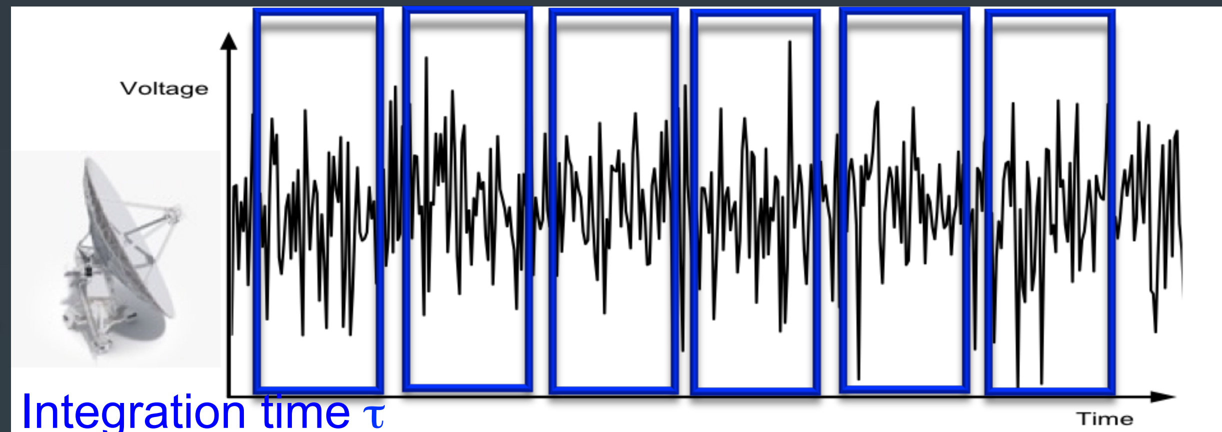
B.Klein MMIC 2010

FPGA – Field-Programmable Gate Array

recap what we measure in a single dish



heterodyne receiver

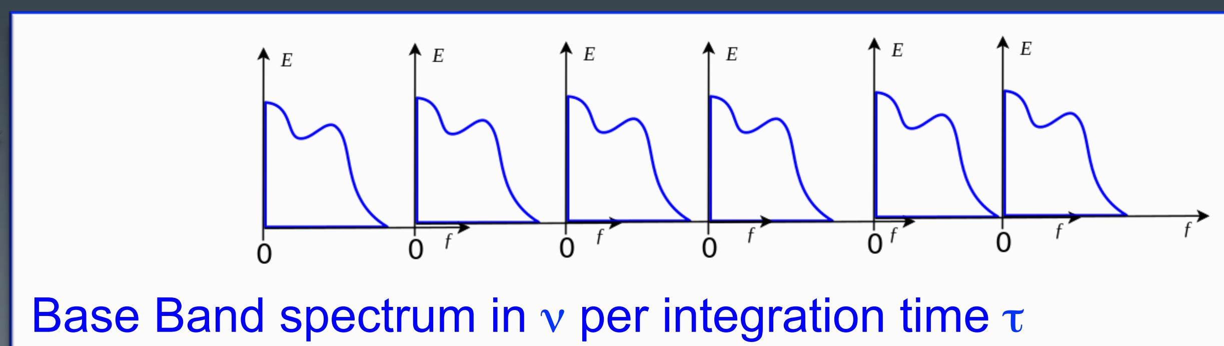


Fourier Transformation

$$F_f(\nu) = \int f(\tau) e^{-2\pi i \nu \tau} d\tau$$

$$F_f(\tau) = \int f(\nu) e^{2\pi i \nu \tau} d\nu$$

Observing time



note usually the integration time will be defined as t

how to get $P(\nu) = |U^2(\nu)|$
- approach 2 -

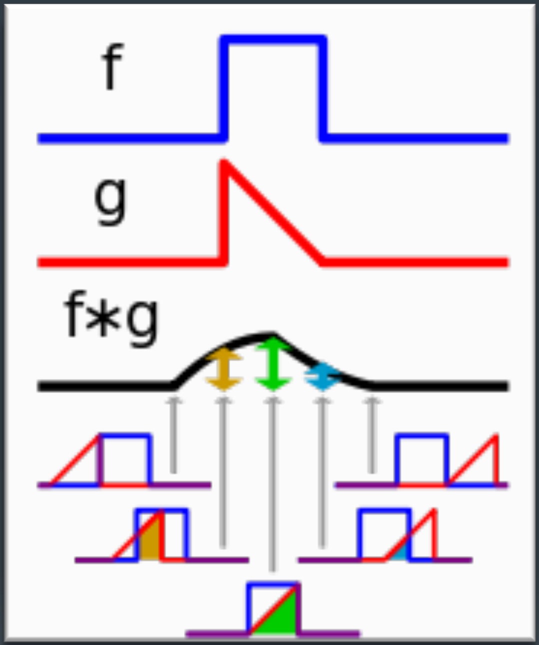
Theoretically

auto correlation function

$$R(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{+T} U(t)\, U(t+\tau)\, dt$$

$$P(\nu) \quad = \quad \int_{-\infty}^{+\infty} R(\tau)\, e^{-i\, 2\pi\, \nu\, \tau}\, d\tau$$

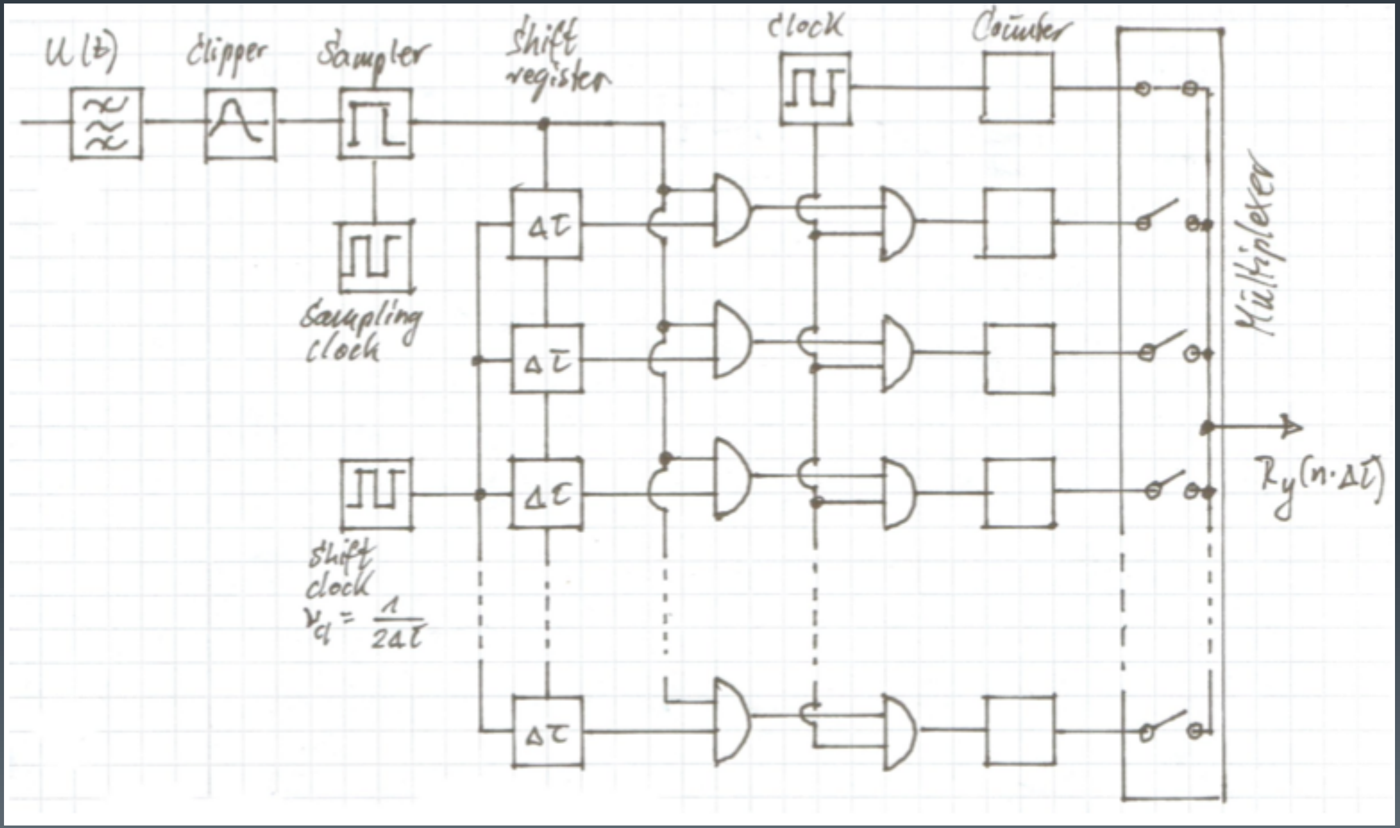
Convolution



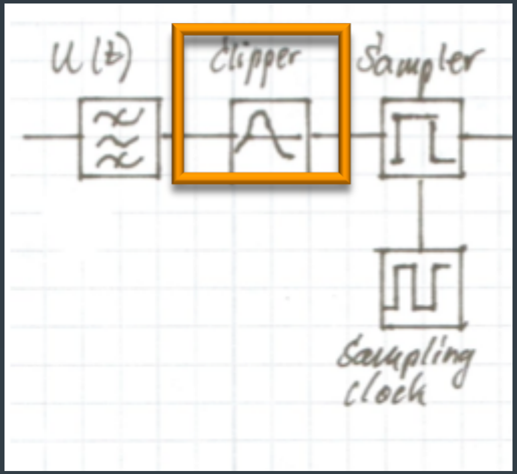
$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

how to get $P(\nu) = |U^2(\nu)|$ - approach 2 -

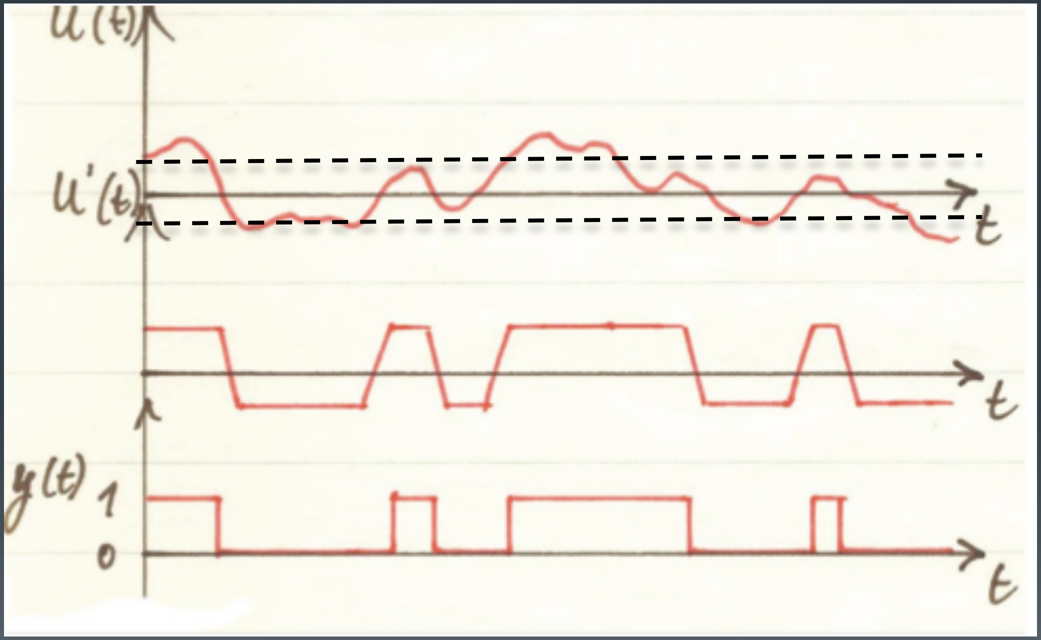
old style hardware to shift the data



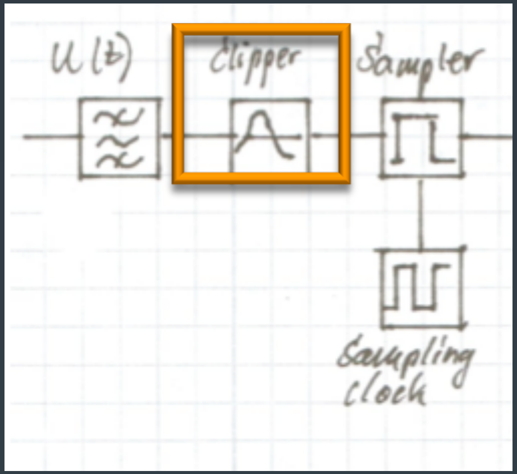
signal processing

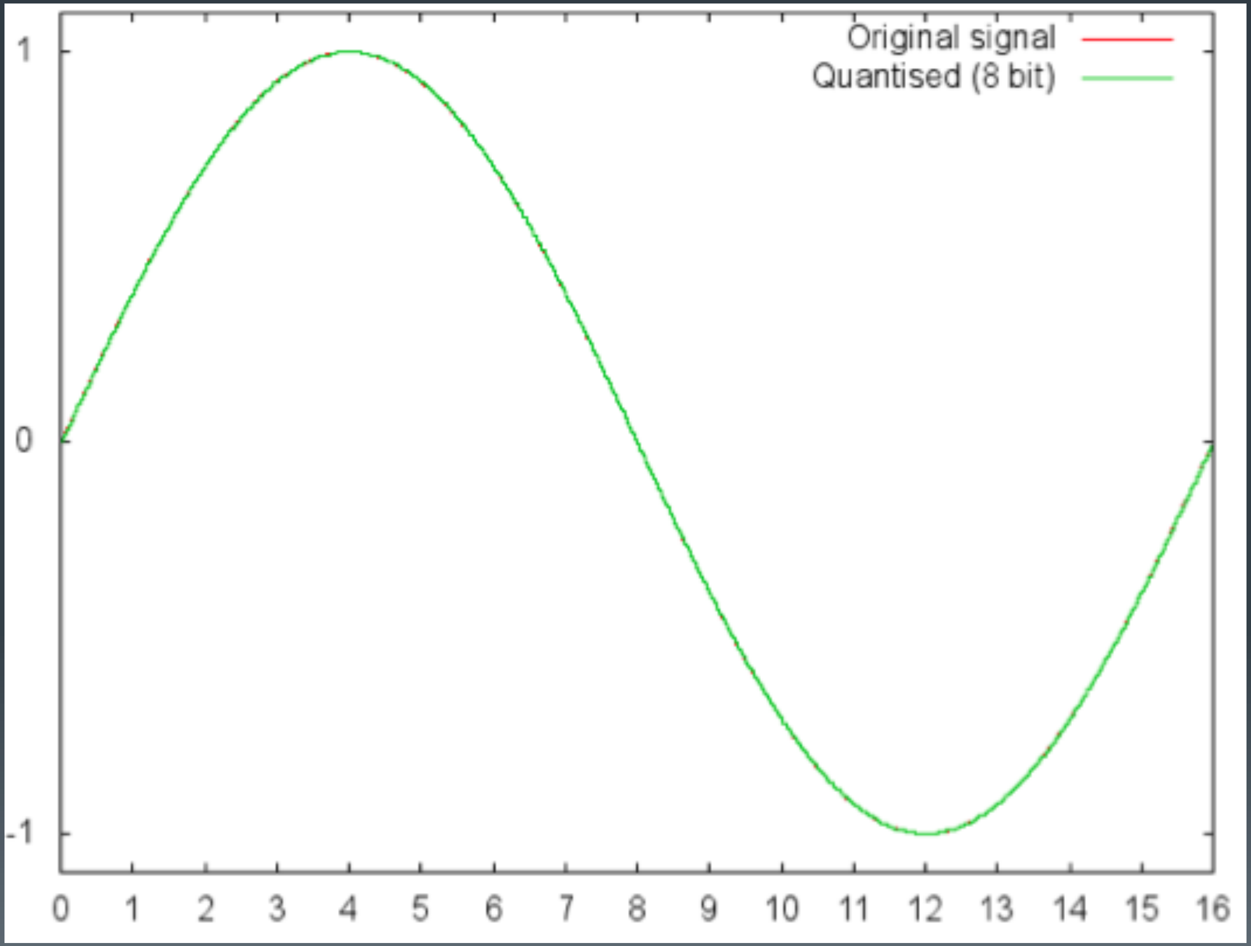


clipper - quantisation 1 bit (0 or 1)

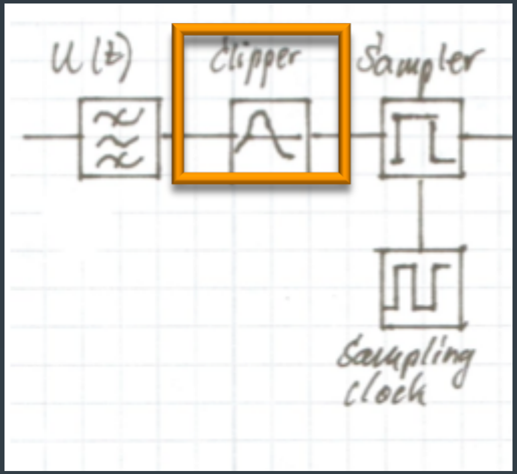


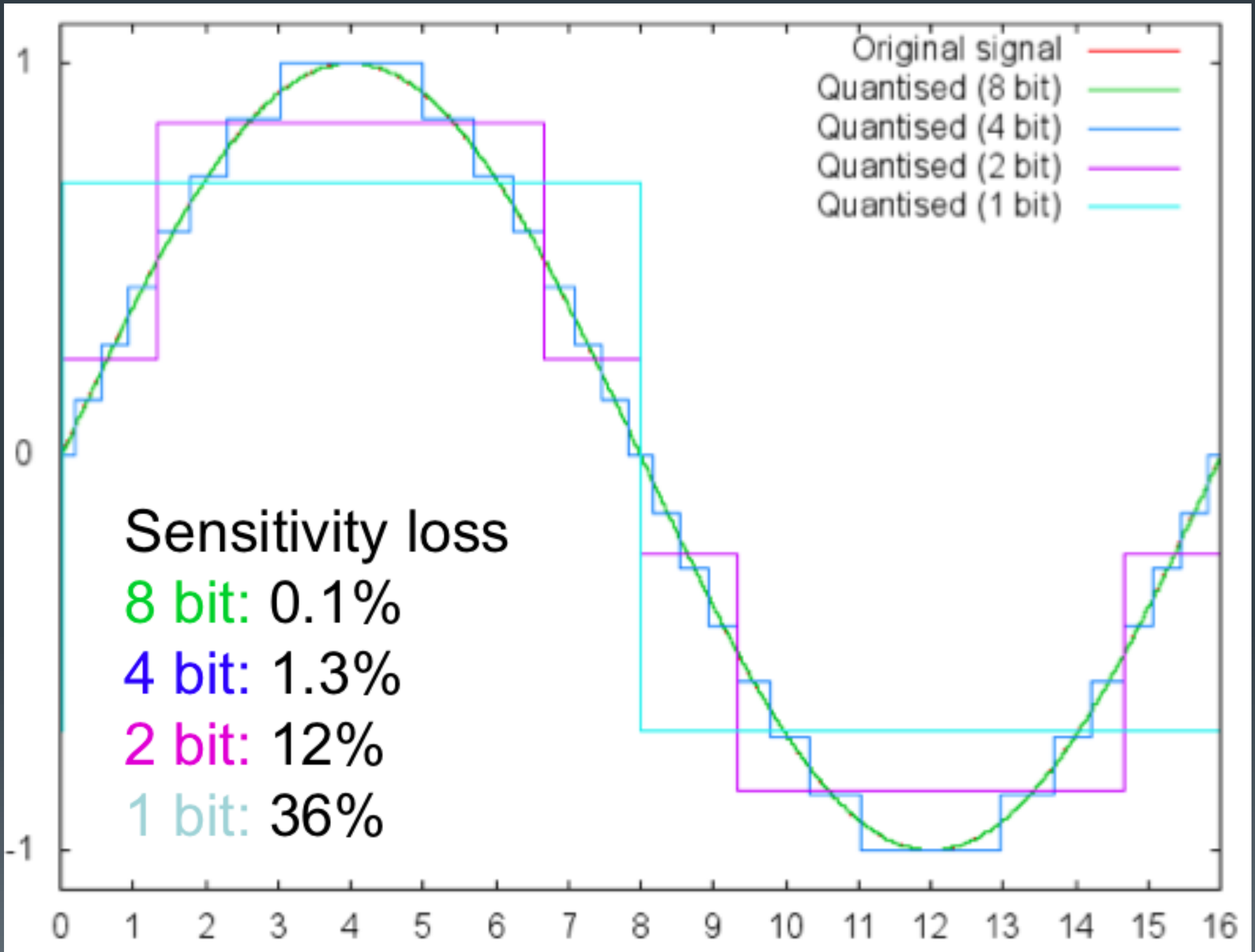
signal processing

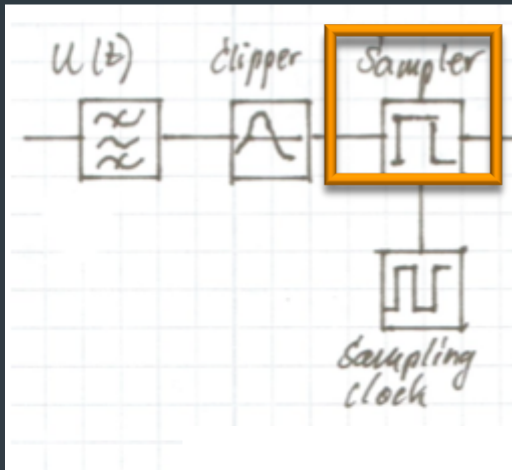




signal processing





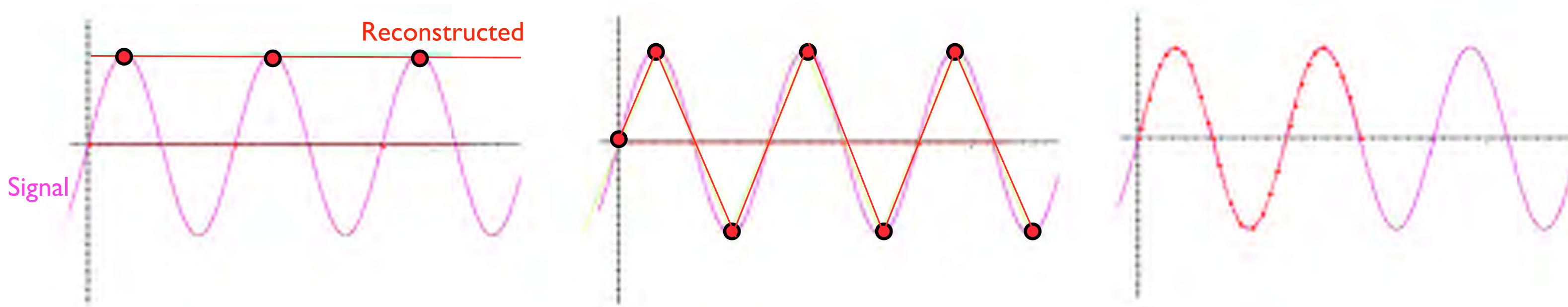


signal processing

If we sample once per cycle time (period) we would consider the signal to have a constant amplitude.

If we sample twice per cycle time (period) we get a saw-tooth wave that is becoming a good approximation to a sinusoid.

For lossless digitisation we must sample the signal at least twice per cycle time.

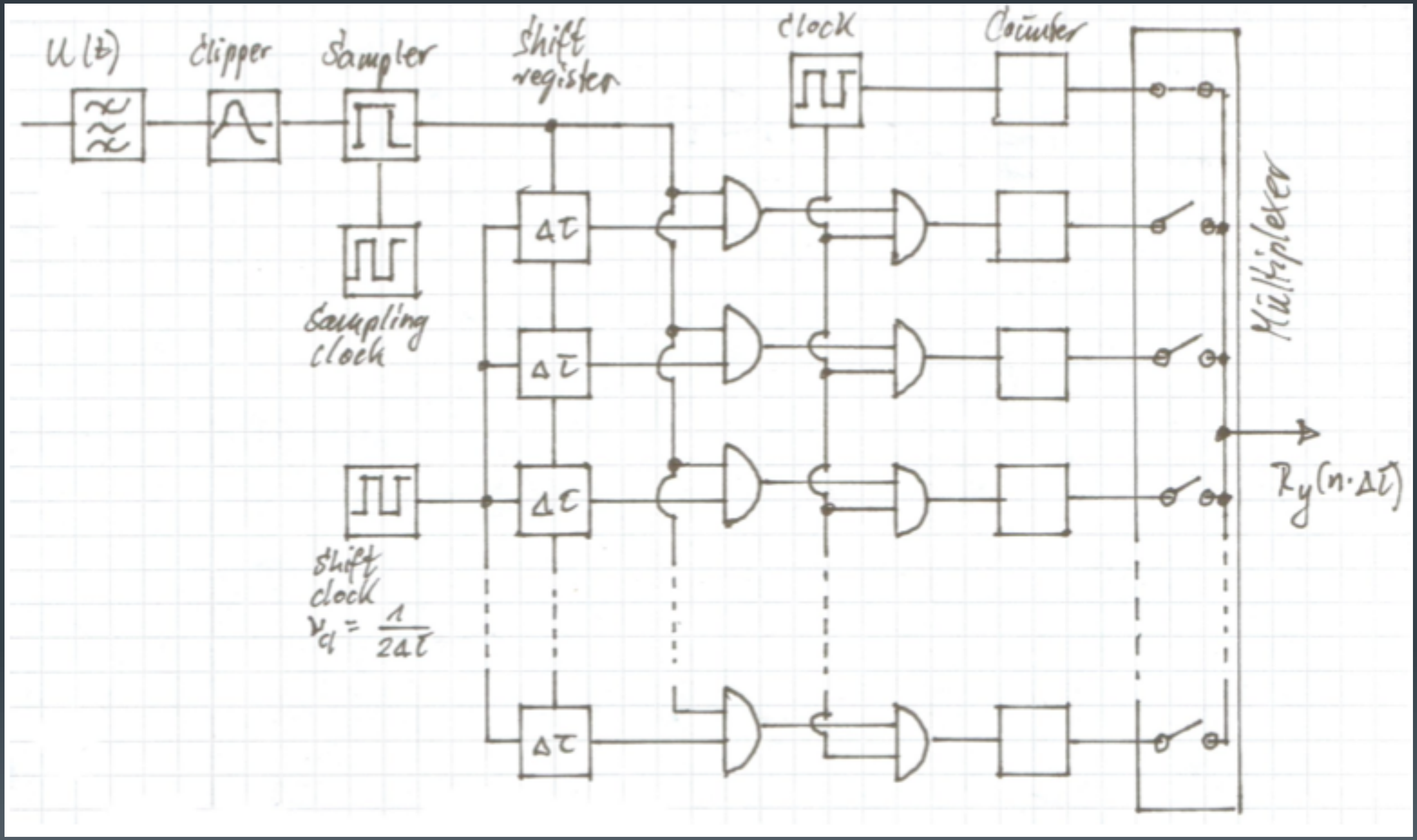


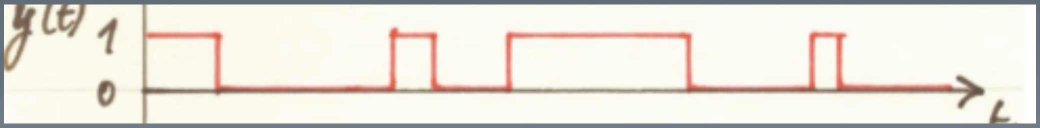
Nyquist's sampling theorem states that for a limited bandwidth signal with maximum frequency f_{max}, the equally spaced sampling frequency f_s must be greater than twice the maximum frequency f_{max}, i.e. $f_s > 2\ f_{max}$ in order for the signal to be uniquely reconstructed without aliasing.

The frequency $2f_{max}$ is called the Nyquist sampling rate.

e.g. *If a reciever system provides a baseband signal of 20 MHz, the signal must be sampled 40E6 times per second.*

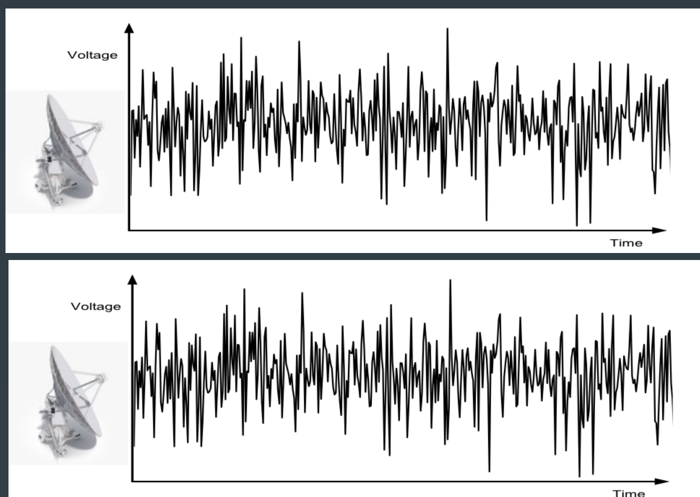
how to get $P(\nu) = |U^2(\nu)|$

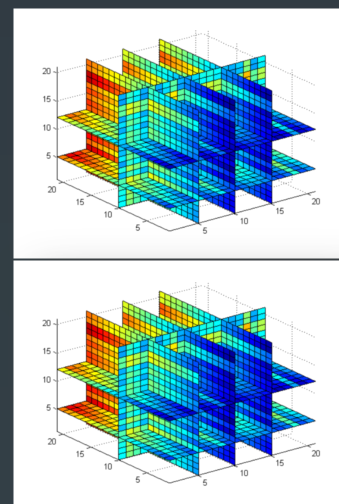


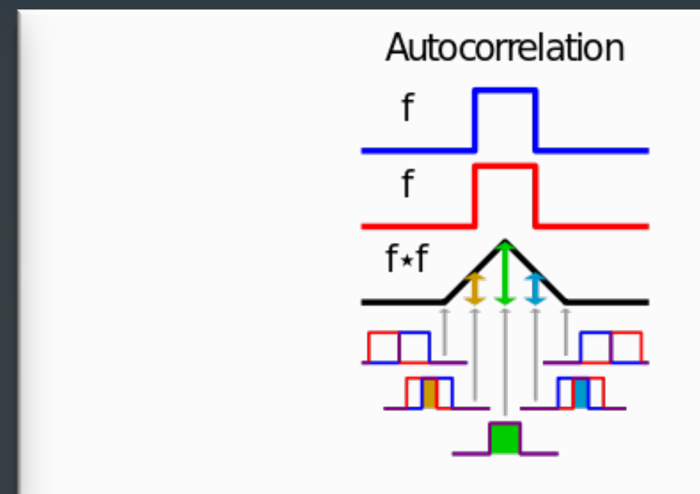


auto correlation

digital data

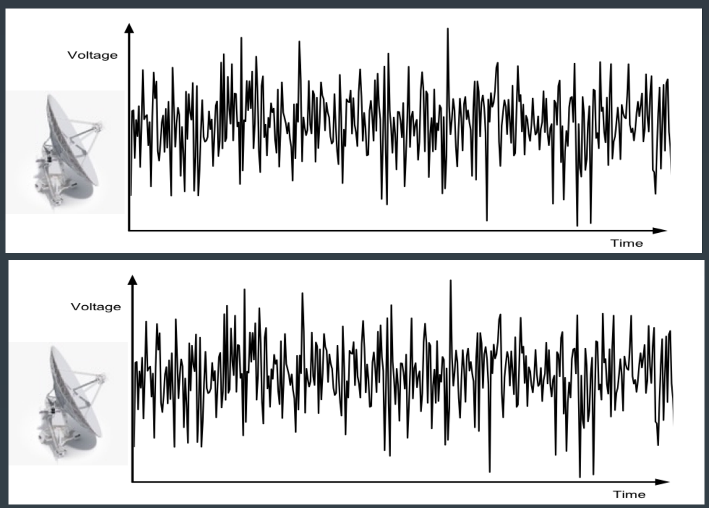


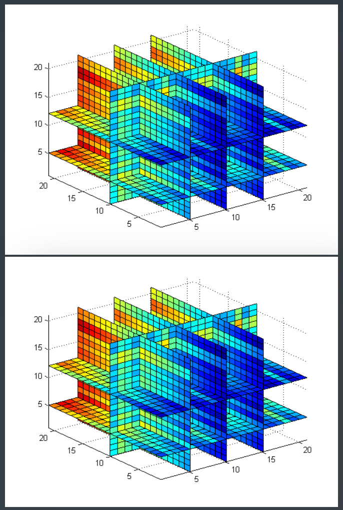


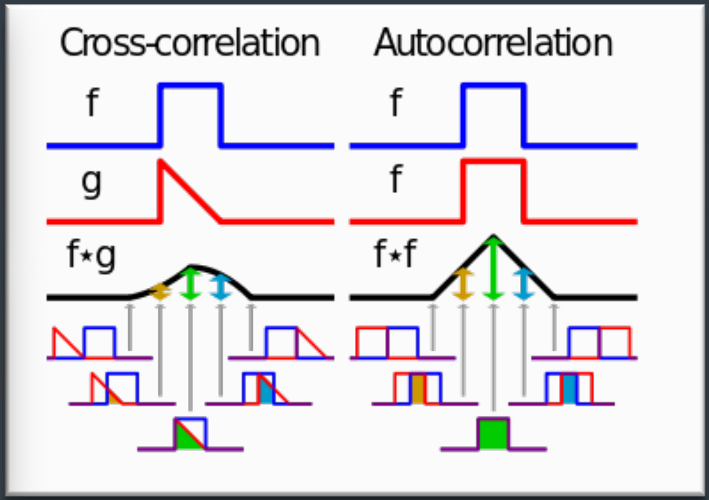


~~auto~~ correlation
cross

digital data

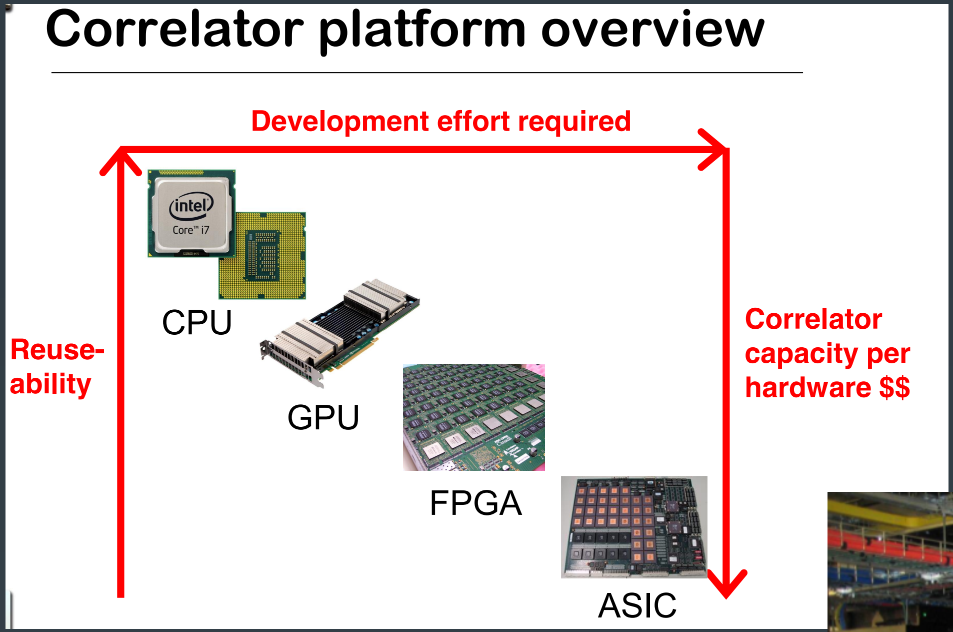


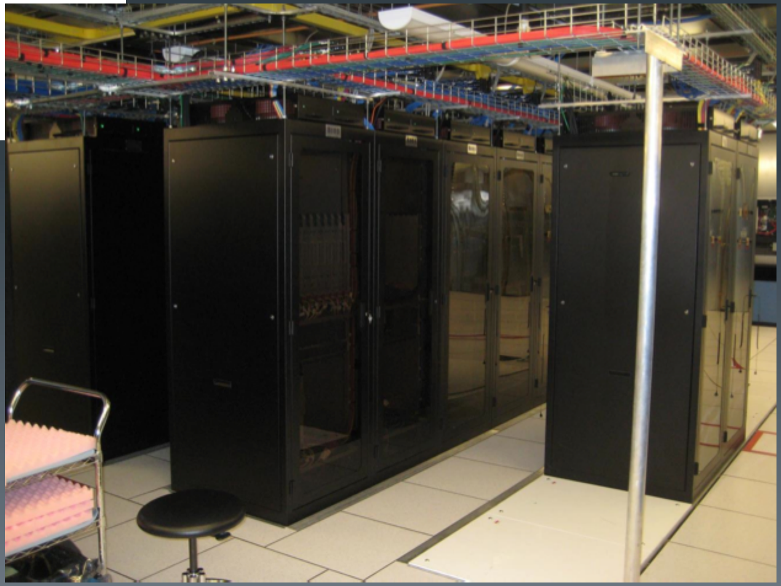




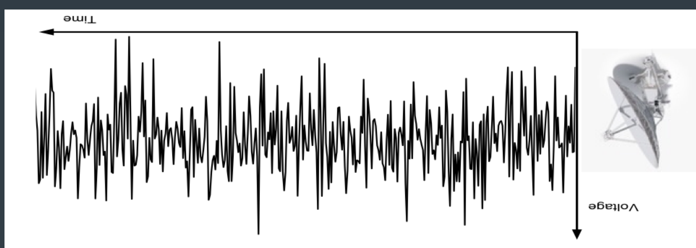
using the signal from different antennas
we build an interferometer

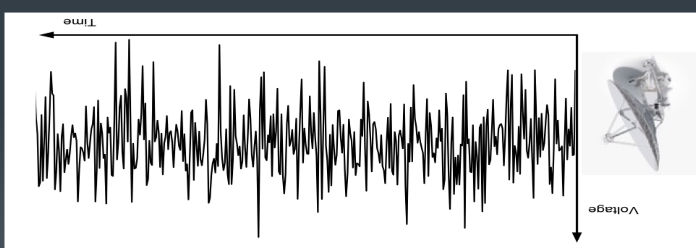
Correlator

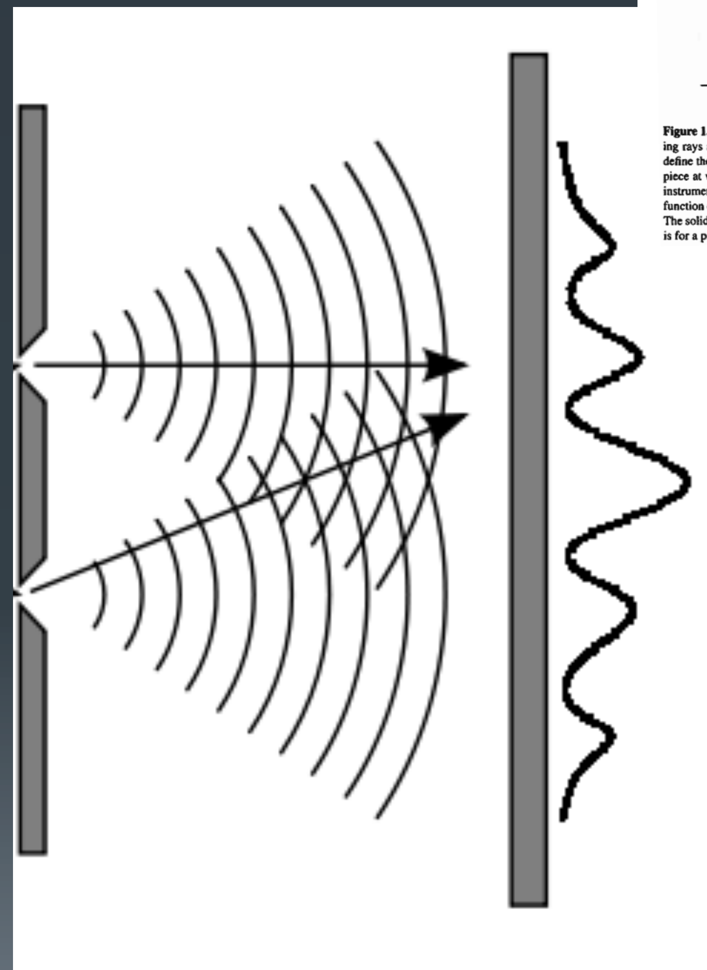

Correlator platform overview
Development effort required
Reuse-
ability
CPU
GPU
FPGA
ASIC
Correlator
capacity per
hardware $$



Young's slit experiment







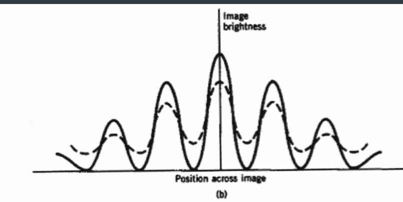
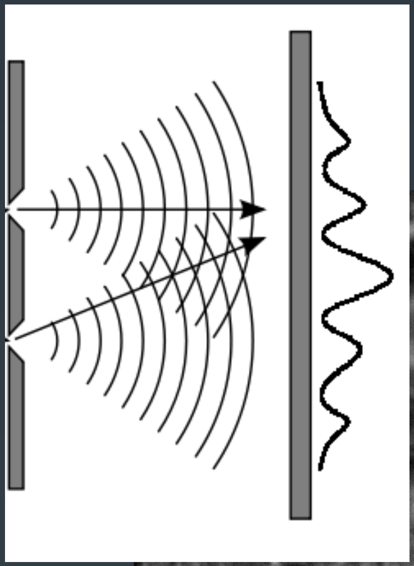


Figure 1.4 (a) Schematic diagram of the Michelson–Pease stellar interferometer. The incoming rays are guided into the telescope aperture by mirrors m_1 to m_4, of which the outer pair define the two apertures of the interferometer. Rays a_1 and b_1 traverse equal paths to the eyepiece at which the image is formed, but rays a_2 and b_2, which approach at an angle θ to the instrumental axis, traverse paths that differ by a distance Δ. (b) The intensity of the image as a function of position angle in a direction parallel to the spacing of the interferometer apertures. The solid line shows the fringe profiles for an unresolved star ($\mathcal{V}_M = 1.0$), and the broken line is for a partially resolved star for which $\mathcal{V}_M = 0.5$.

solid line unresolved

dashed line resolved

aperture synthesis



mix the signal from all the telescope that they are in phase