Principles of Interferometry

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IMPRS Black Board Lectures 2014

acknowledgement

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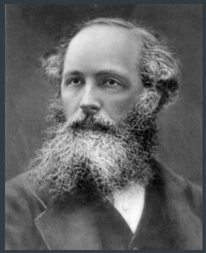
- NRAO Summer School lectures
- Phil Diamond ICRAR
- Ger de Bruyn SKADS workshop

Lecture 2

- radio astronomical terms and definitions
- antenna temperature
- single dish telescope type
- single dish telescope beams
- sensitivity
- basic calibration

Maxwell Equations



$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}.$$

vacuum equations

Poynting vector
energy flux of an electro-magnetic wave

S = c/4pi **E** x **B**   [W m^{-2}]

vector waves

The electric field vector of a monochromatic electromagnetic plane wave is perpendicular to the direction of the field. The vector can be given as a sum of two orthogonal components:

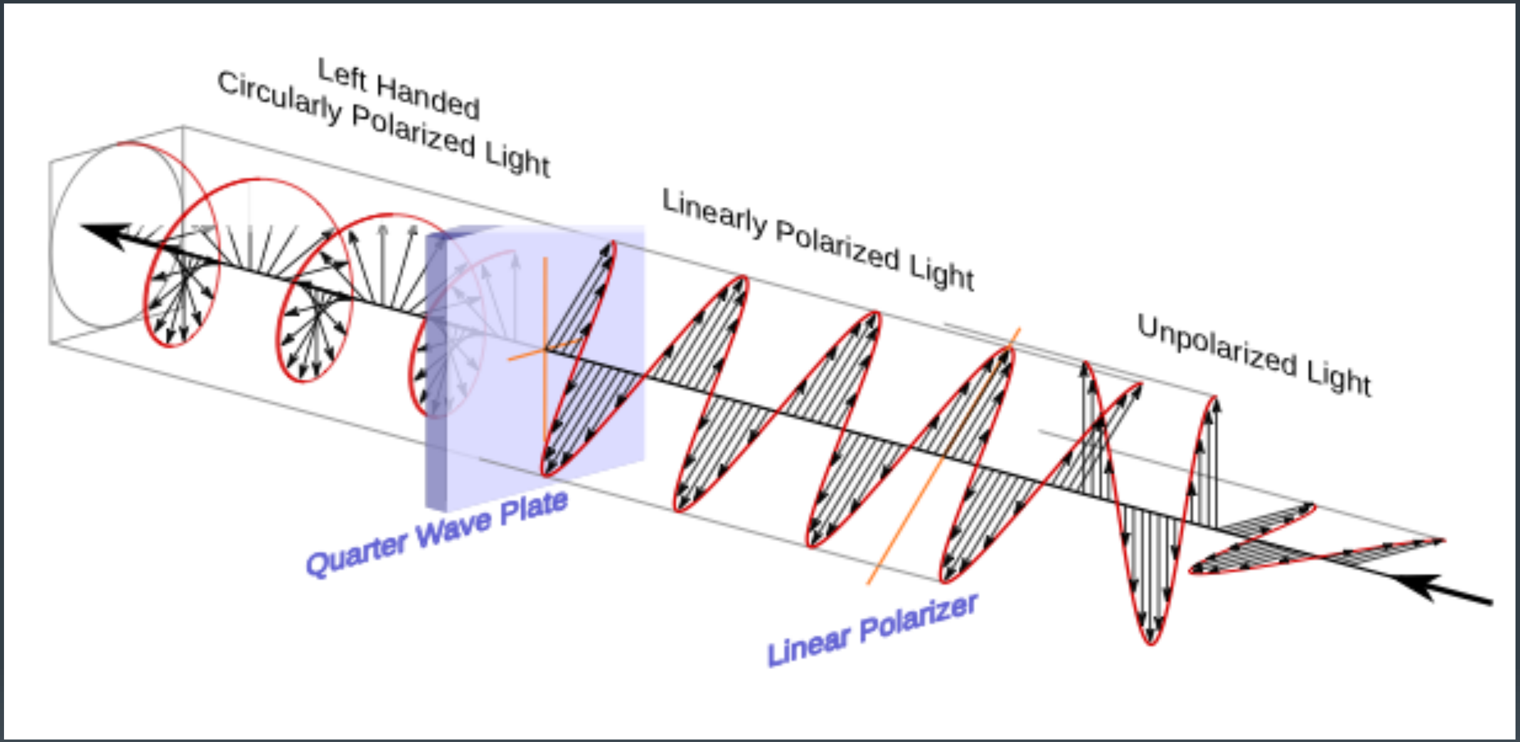
$$
\begin{aligned}
E_x &= E_1 \cos(kz - \omega t + \delta_1) \\
E_y &= E_2 \cos(kz - \omega t + \delta_2) \\
E_z &= 0, \qquad (2)
\end{aligned}
$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$. The equation (2) describes a helix on the surface of a cylinder.
The cross section is

$$
\left(\frac{E_x}{E_1}\right)^2 + \left(\frac{E_y}{E_2}\right)^2 - 2\frac{E_x}{E_1}\frac{E_y}{E_2}\cos\delta = \sin^2\delta, \qquad (3)
$$

where $\delta = \delta_1 - \delta_2$.

polarised wave



$$S_0 = I = E_1^2 + E_2^2$$
$$S_1 = Q = E_1^2 - E_2^2$$
$$S_2 = U = 2E_1E_2\cos\delta$$
$$S_3 = V = 2E_1E_2\sin\delta$$

Stokes parameter

$$S_0^2 \geq S_1^2 + S_2^2 + S_3^2$$
$$I^2 \geq Q^2 + U^2 + V^2.$$

degree of polarisation

$$p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}$$

brightness temperature black body

- Properties of "black-body radiation" (you should all be familiar with this!)
  - functional form is called the "Planck function":

$$B_\nu(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1} \qquad (1)$$

Units of spectral energy density are Watts per Hz per sq. metre per steradian

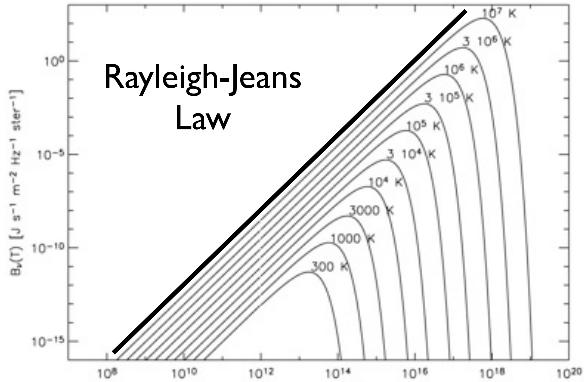
Radio photons are pretty wimpy: $h\nu/kT << 1$ $\qquad e^{h\nu/kT} \sim 1 + h\nu/kT + ...$

$$\Longrightarrow B_\nu(T) = 2kT\nu^2/c^2 \qquad (2)$$

- Eqn(2) is known as the Raleigh-Jeans law i.e. at low frequencies the intensity increases with the square of the frequency.

  Note that the R-J law holds all the way through the radio regime for any reasonable temperature

  k is boltzmann's constant = 1.38E-23 $m^2\,kg\,s^{-2}\,K^{-1}$



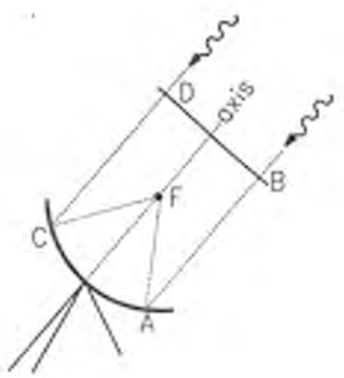
Radio imaging

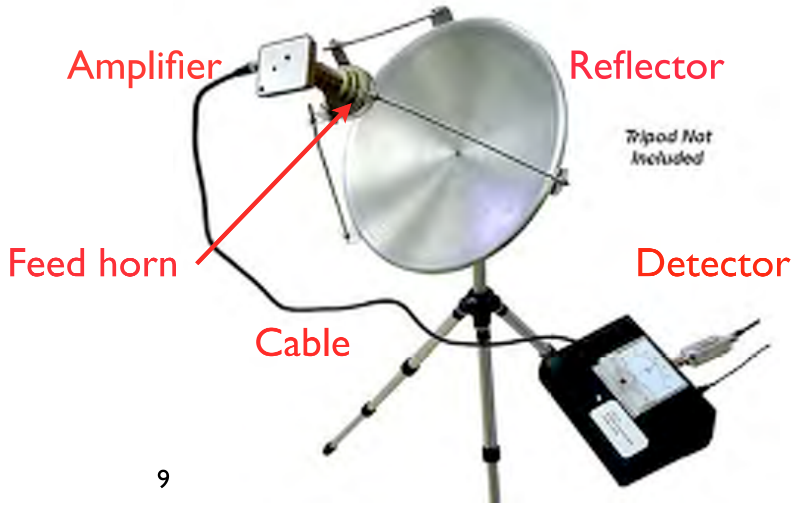
Radio photons are too wimpy to do very much - we cannot usually detect individual photons

- e.g. optical photons of 600 nanometre => 2 eV or 20000 Kelvin (hν/kT)
- e.g. radio photons of 1 metre => 0.000001 eV or 0.012 Kelvin

➡ Photon counting in the radio is not usually an option, we must think classically in terms of measuring the source electric field etc.

i.e. measure the voltage oscillations induced in a conductor (antenna) by the incoming EM-wave. Example:



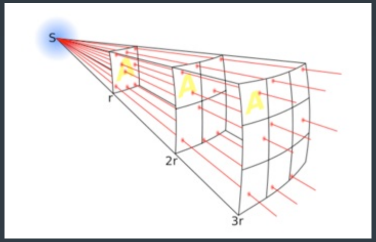


Nyquist law

A resistor (even without current) at temperature T produce noise power $P_\nu \, d\nu = kT \, d\nu$

assume one could connect the resistor to the telescope without any loss one would measure the telescope temperature

surface brightness & flux density



EM power in bandwidth $\delta\nu$ from solid angle $\delta\Omega$ intercepted by surface δA is:

$$\delta W = I_\nu \delta\Omega \delta A \delta\nu$$

Defines surface brightness $\boldsymbol{I_\nu}$ (W m^{-2} Hz^{-1} sr^{-1} ; aka specific intensity)

Flux density $\boldsymbol{S_\nu}$ (W m^{-2} Hz^{-1}) – integrate brightness over solid angle of source

$$S_\nu = \int_{\Omega_s} I_\nu d\Omega$$

Convenient unit – the **Jansky** ➔ 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Note: $S_\nu = L_\nu / 4\pi d^2$ ie. distance dependent

$\Omega \propto 1/d^2 \Rightarrow I_\nu \propto S_\nu / \Omega$ ie. distance independent

brightness temperature - source

Many astronomical sources DO NOT emit as blackbodies!

However….

Brightness temperature (T_B) of a source is defined as the temperature of a blackbody with the same surface brightness at a given frequency:

$$I_\nu = \frac{2k\nu^2 T_B}{c^2}$$

This implies that the flux density $S_\nu = \int_{\Omega_s} I_\nu d\Omega = \frac{2k\nu^2}{c^2} \int T_B d\Omega$

What does a Radio Telescope detect

Recall : $$\delta W = I_\nu \delta\Omega \delta A \delta\nu$$

Telescope of effective area A_e receives power P_{rec} per unit frequency from an unpolarized source but is only sensitive to one mode of polarization:

$$P_{rec} = \frac{1}{2} I_\nu A_e \delta\Omega$$

Telescope is sensitive to radiation from more than one direction with *relative* sensitivity given by the normalized antenna pattern $P_N(\theta,\varphi)$:

$$P_{rec} = \frac{1}{2} A_e \int_{4\pi} I_\nu(\theta,\varphi) P_N(\theta,\varphi) d\Omega$$

brightness temperature - telescope

In general surface brightness is position dependent, ie. $I_\nu = I_\nu(\theta,\phi)$

$$I_\nu(\theta,\varphi) = \frac{2k\nu^2 T(\theta,\varphi)}{c^2}$$

(if I_ν described by a blackbody in the Rayleigh-Jeans limit; $h\nu/kT << 1$)

Back to flux:

$$S_\nu = \int_{\Omega_s} I_\nu(\theta,\varphi)d\Omega \quad = \frac{2k\nu^2}{c^2}\int T(\theta,\varphi)d\Omega$$

In general, a radio telescope maps the *temperature distribution of the sky*

measurement

$$S_\nu = \int_{\Omega_s} I_\nu(\theta,\varphi)d\Omega \quad = \frac{2k\nu^2}{c^2}\int T(\theta,\varphi)d\Omega$$

$$S_\nu = \int_{\Omega_s} I_\nu d\Omega = \frac{2k\nu^2}{c^2}\int T_B d\Omega$$

$$P_{rec} = \frac{A_e}{2}\int_{4\pi} I_\nu(\theta,\varphi)P_N(\theta,\varphi)\, d\Omega$$

$$\therefore T_A = \frac{A_e}{2k}\int_{4\pi} I_\nu(\theta,\varphi)P_N(\theta,\varphi)\, d\Omega$$

Antenna temperature is what is observed by the radio telescope.

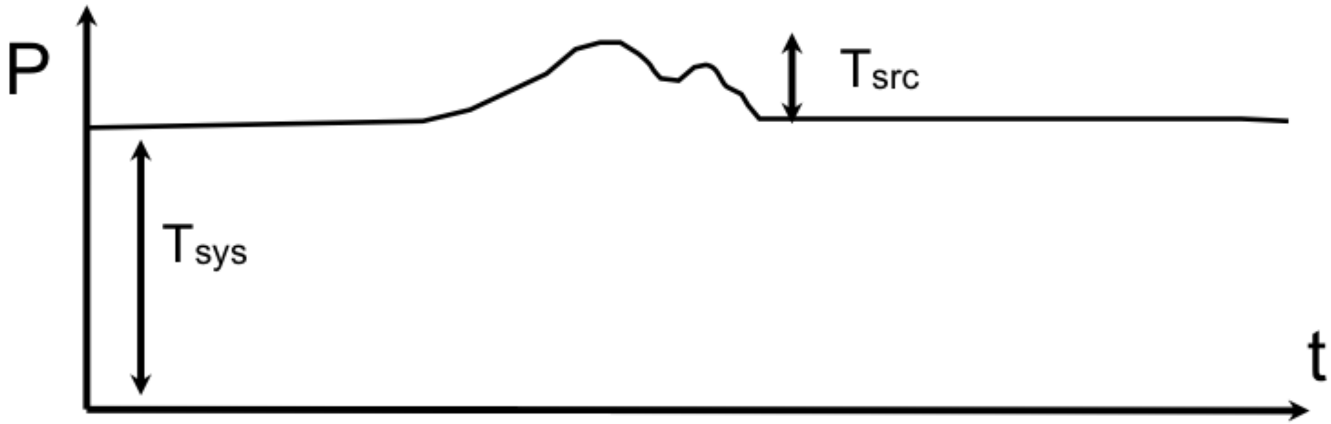
A "convolution" of sky brightness with the beam pattern

It is an inversion problem to determine the source temperature distribution.

$$S_\nu = \frac{2k}{A_{eff}}T_A$$

radio "imaging"

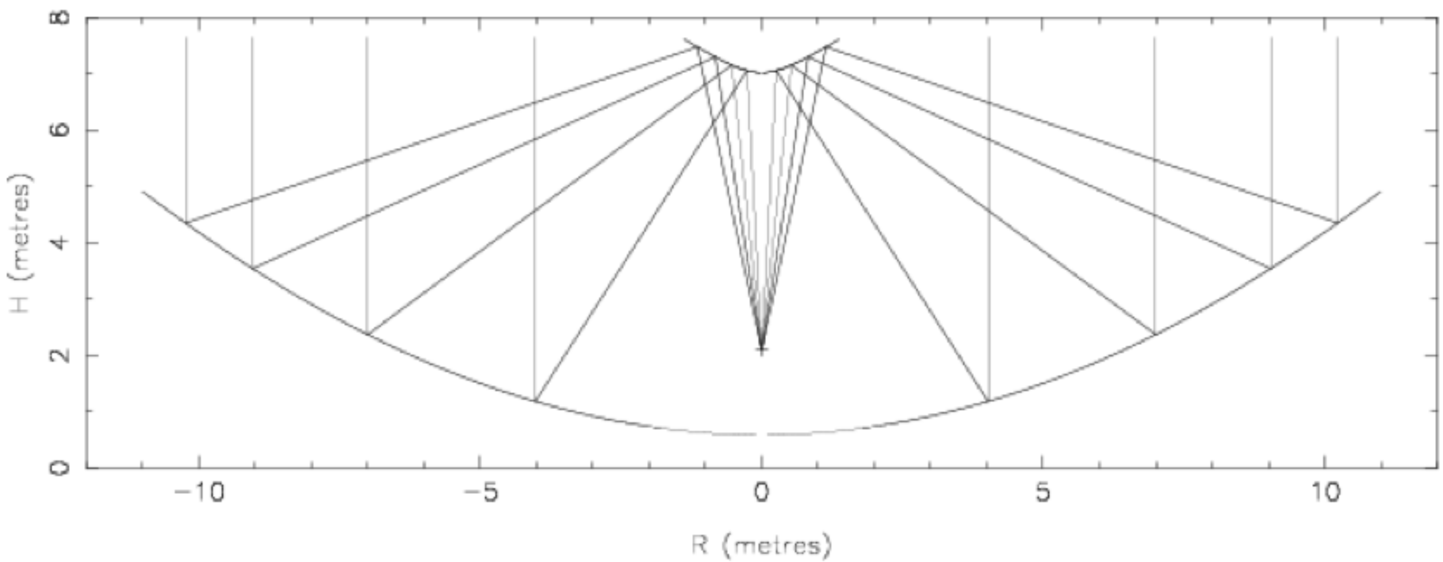
Imaging of the sky with a single-dish can be achieved by letting the source drift across the telescope beam and measuring the power received as a function of time. This provides a 1-D cut across the source intensity. Usually, the area of interest is measured at least twice, in orthogonal directions (sometimes referred to as "basket weaving").



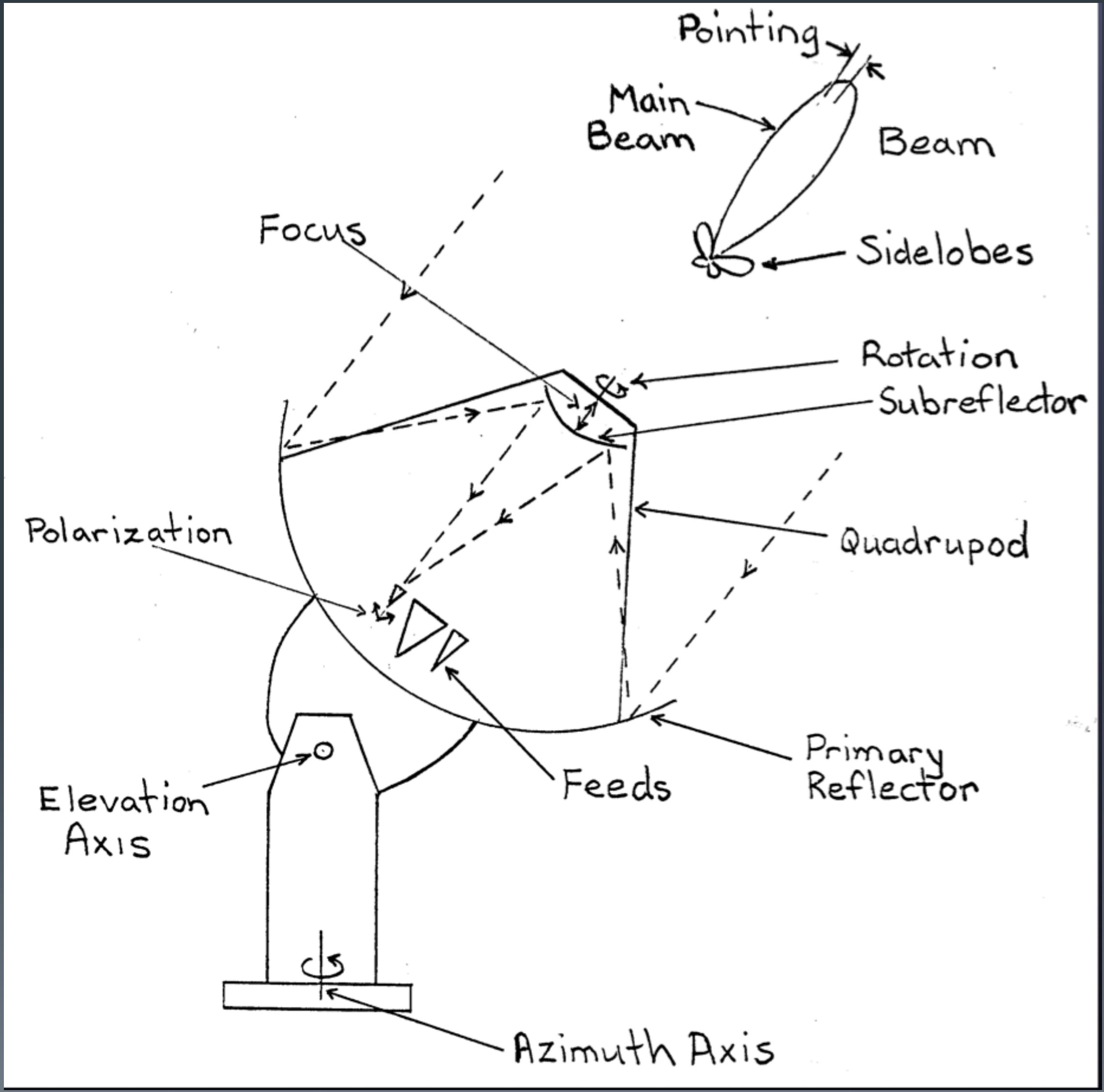
radio telescope

The *antenna* collects the E-field over the aperture at the focus

The *feed horn* at the focus adds the fields together, guides signal to the *front end*

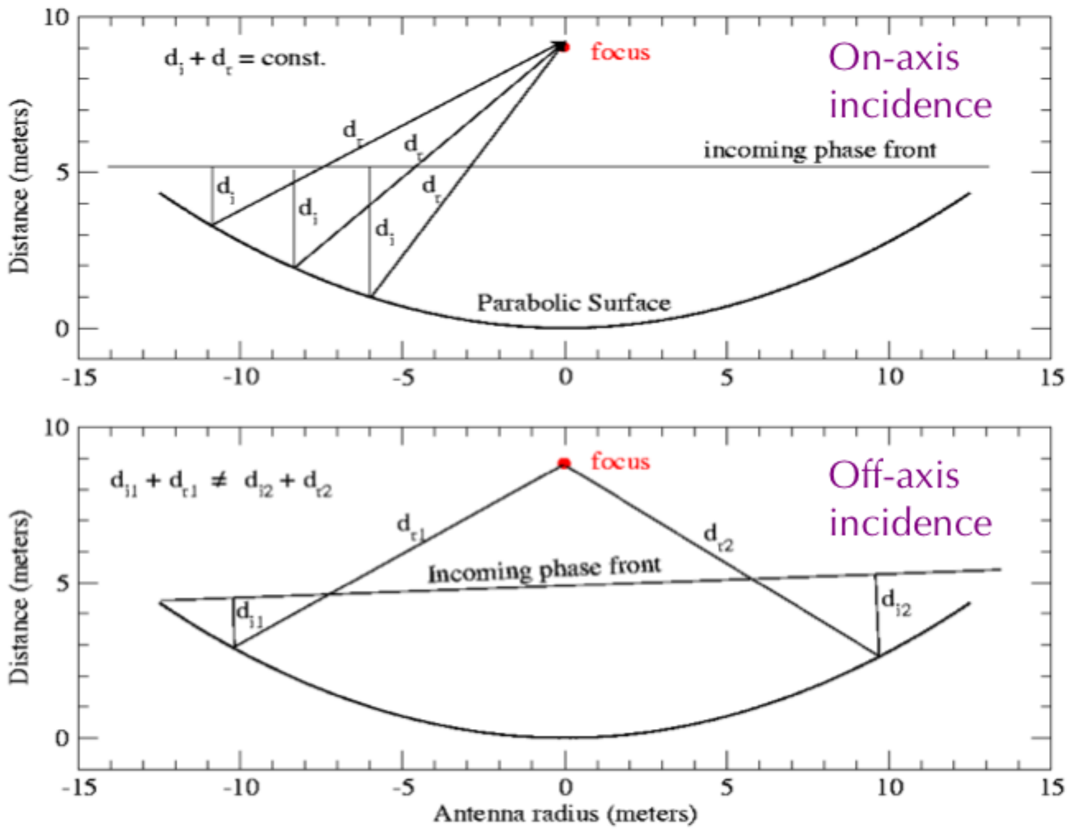


primary antenna key features

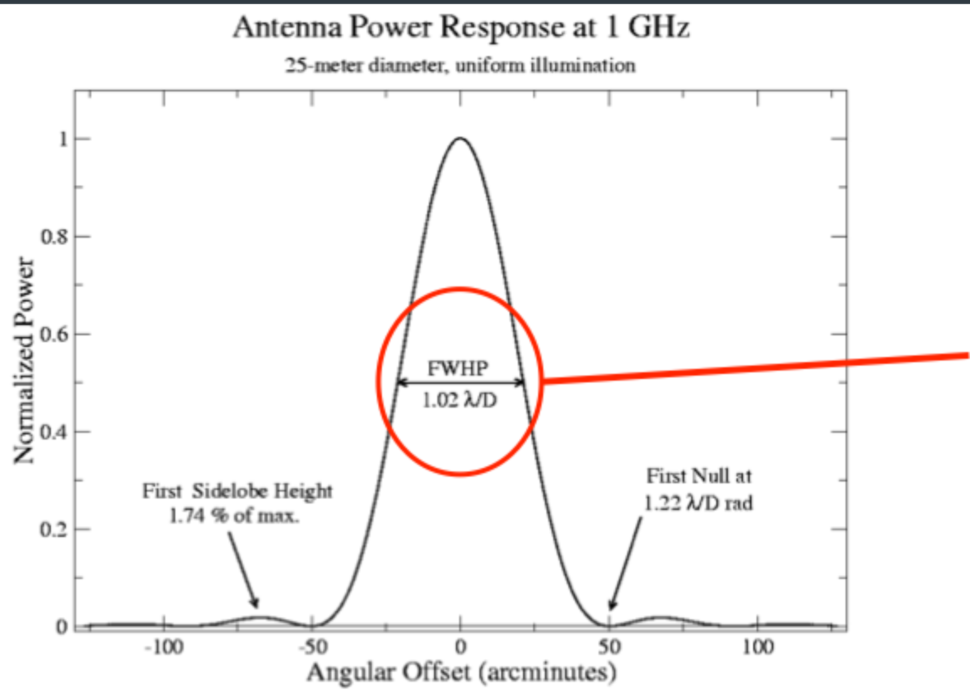


origin of the beam pattern

- Antenna response is a coherent phase summation of the E-field at the focus
- First null occurs at the angle where one extra wavelength of path is added across the full aperture width, i.e., $\theta \sim \lambda/D$



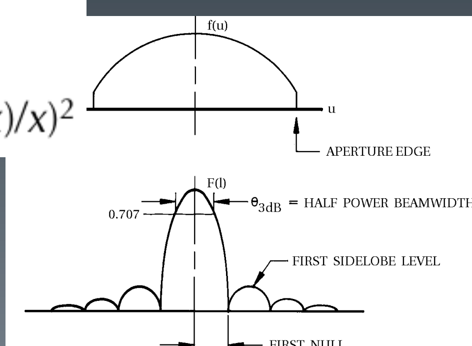
antenna power pattern



Defines telescope resolution

- The voltage response pattern is the FT of the aperture distribution
- The power response pattern, $P(\theta) \propto V^2(\theta)$, is the FT of the autocorrelation function of the aperture
- for a uniform circle, $V(\theta)$ is $J_1(x)/x$ *and* $P(\theta)$ is the Airy pattern, $(J_1(x)/x)^2$

J_1 Bessel Function



the beam

$P(\theta,\varphi,\nu) = A(\theta,\varphi,\nu)\ I(\theta,\varphi,\nu)\ \Delta\nu\ \Delta\Omega$

effective collecting area $A(\nu,\theta,\varphi)$ [m^2]

on-axis response

$$A_0 = \eta A$$

η = aperture efficiency

Normalized pattern (primary beam)

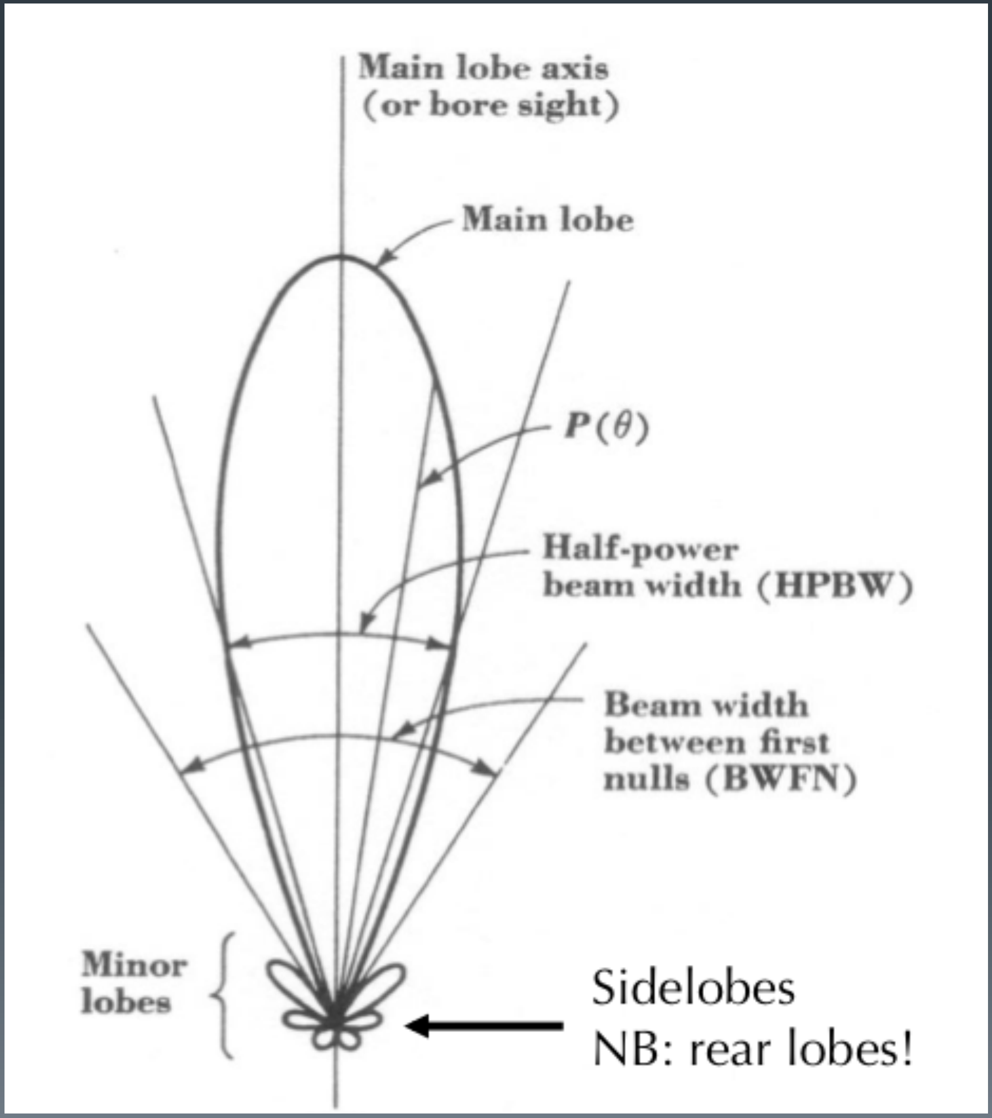
$\boldsymbol{A}(\nu,\theta,\varphi) = A(\nu,\theta,\varphi)/A_0$

Beam solid angle

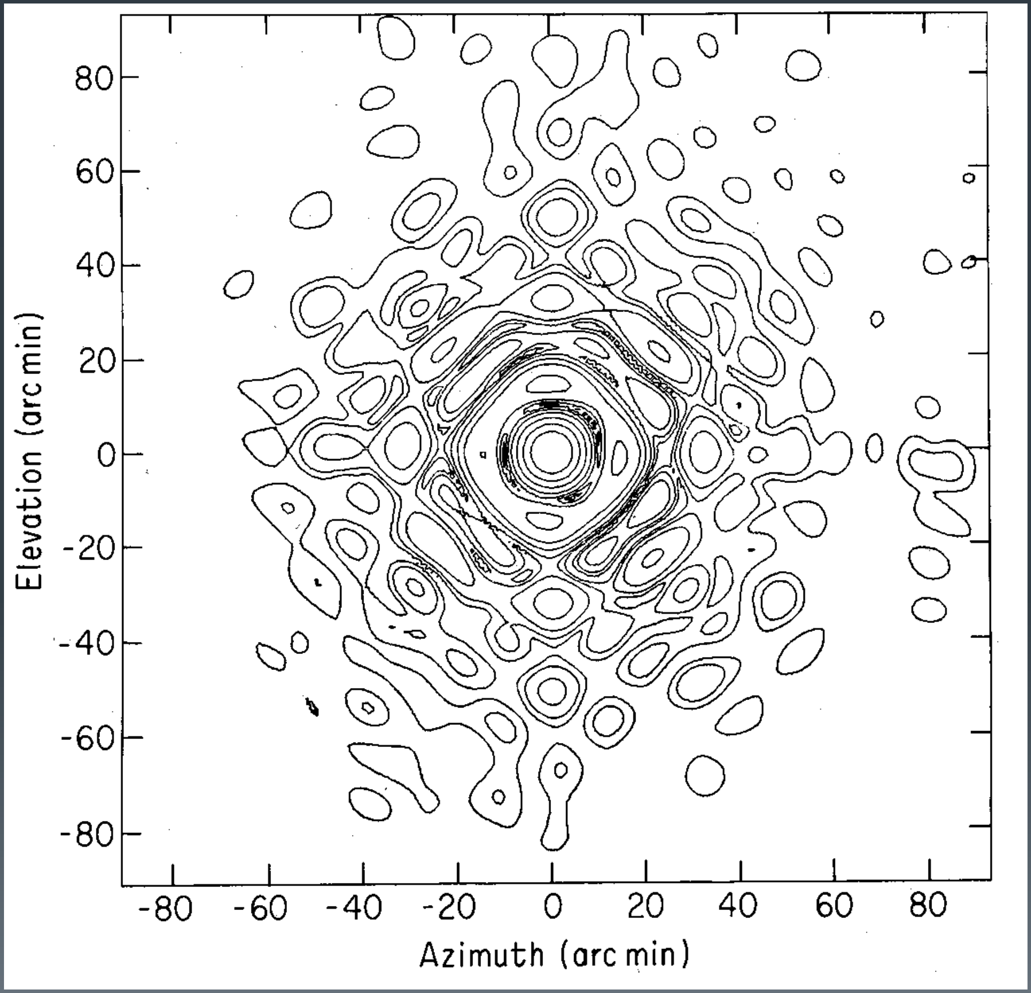
$\Omega_A = \iint \boldsymbol{A}(\nu,\theta,\varphi)\ d\Omega$ all sky

$A_0\ \Omega_A = \lambda^2$

λ = wavelength, ν = frequency



a real beam



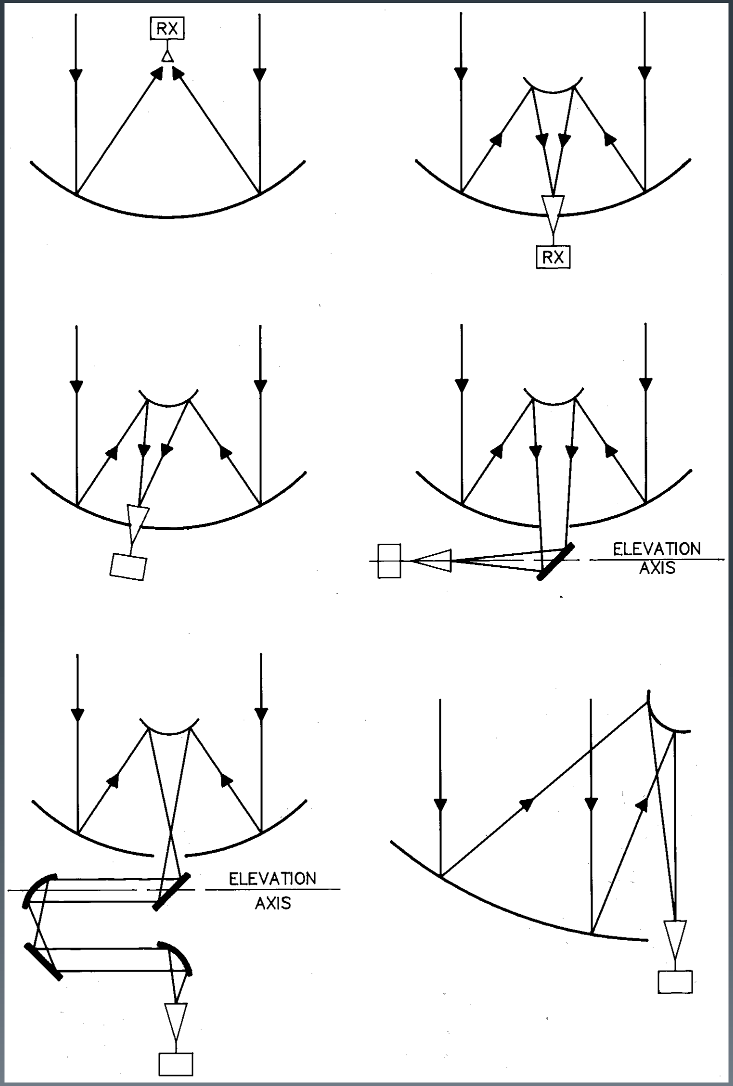
80
60
40
20
0
-20
-40
-60
-80
Elevation (arc min)
-80
-60
-40
-20
0
20
40
60
80
Azimuth (arc min)

reflector types

Prime focus
(GMRT)

Offset Cassegrain
(VLA)

Beam Waveguide
(NRO)

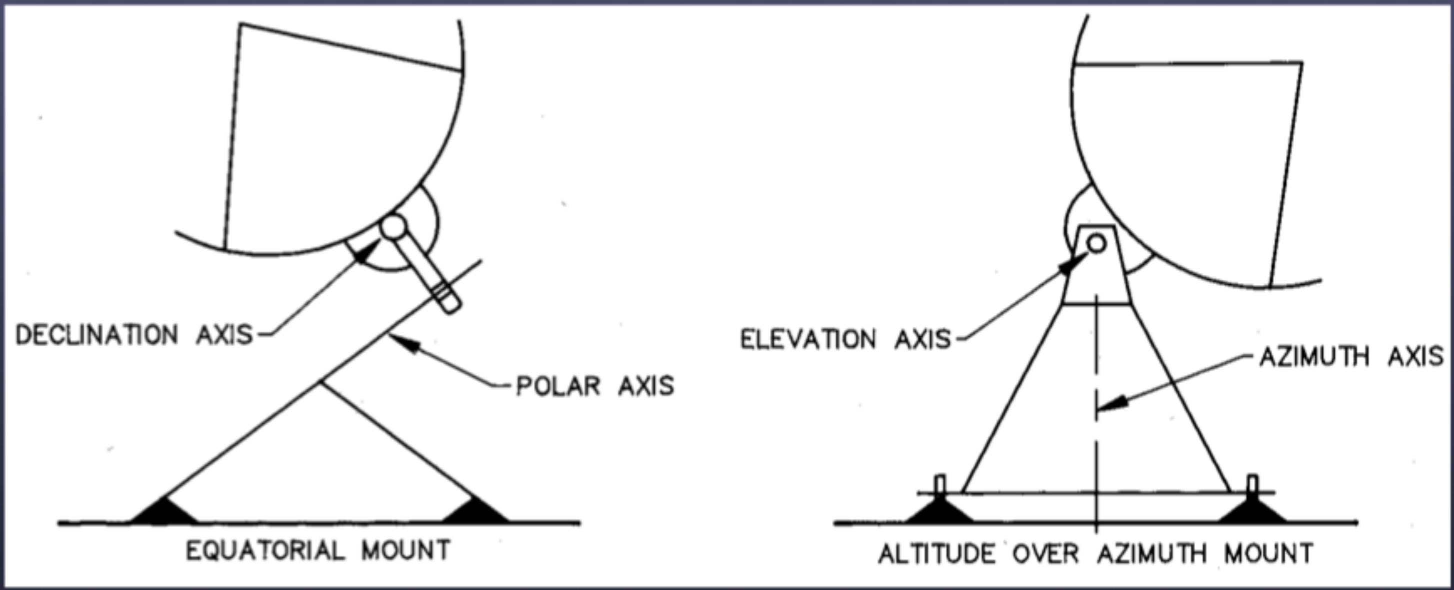


Cassegrain focus
(AT)

Naysmith
(OVRO)

Dual Offset
(ATA)

antenna mount



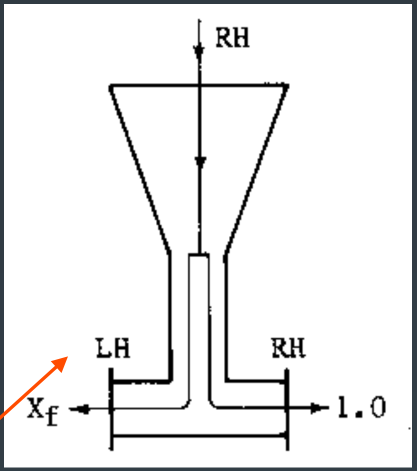
+ Beam does not rotate
+ Better tracking accuracy
− Higher cost
− Poorer gravity performance
− Non-intersecting axis

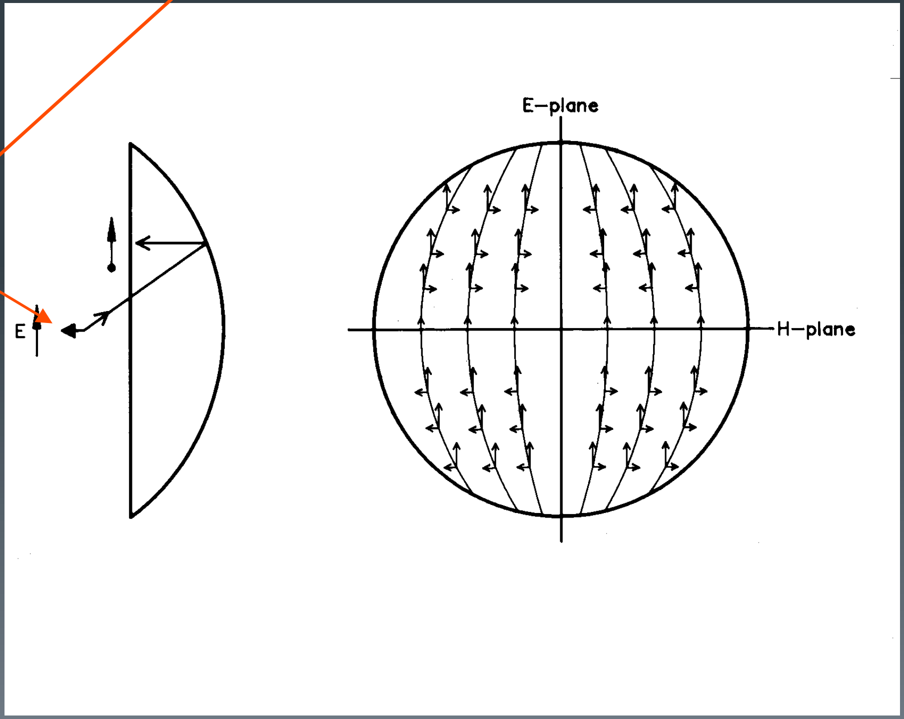
+ Lower cost
+ Better gravity performance
− Beam rotates on the sky

polarisation

Antenna can modify the apparent polarisation properties of the source:

- Symmetry of the optics
- Quality of feed polarisation splitter
- Circularity of feed radiation patterns
- Reflections in the optics
- Curvature of the reflectors
- paralactic angle - mount dependent

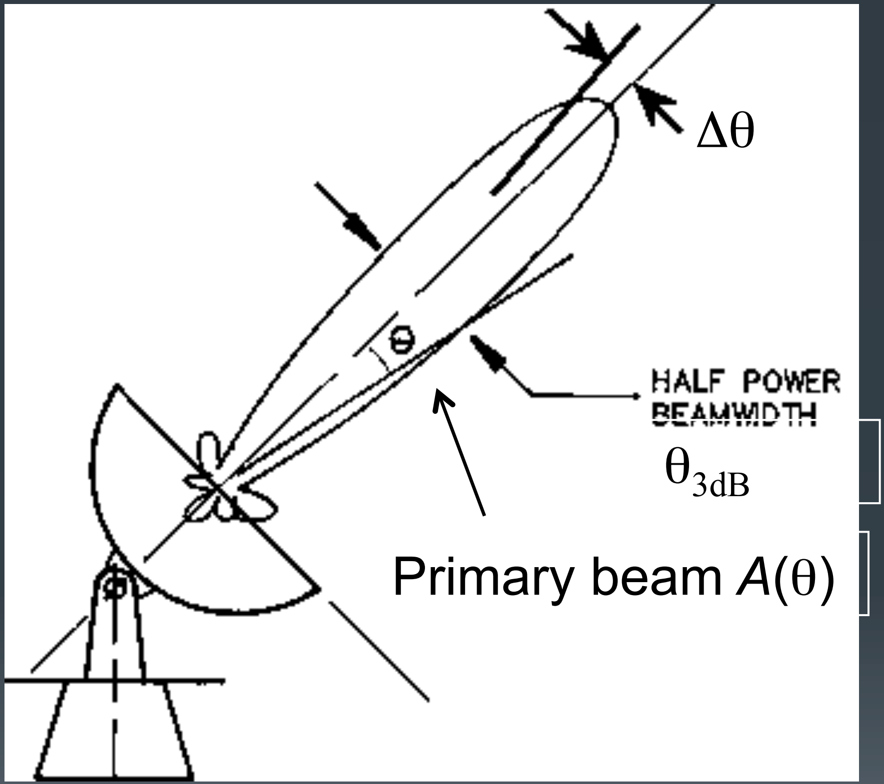




pointing accuracy

Pointing Accuracy
$\Delta\theta$ = rms pointing error

Often $\Delta\theta < \theta_{3dB}/10$ acceptable
Because $\boldsymbol{A}(\theta_{3dB}/10) \sim 0.97$
BUT, at half power point in beam
$\boldsymbol{A}(\theta_{3dB}/2 \pm \theta_{3dB}/10)/\boldsymbol{A}(\theta_{3dB}/2) = \pm 0.3$



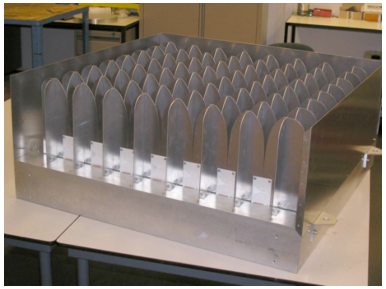
For best VLA pointing use Reference Pointing.
$\Delta\theta$ = 3 arcsec = $\theta_{3dB}/17$ @ 50 GHz

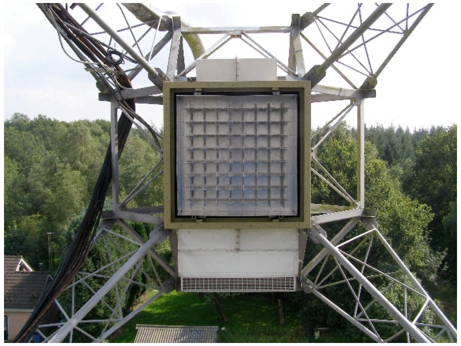
focal plane arrays

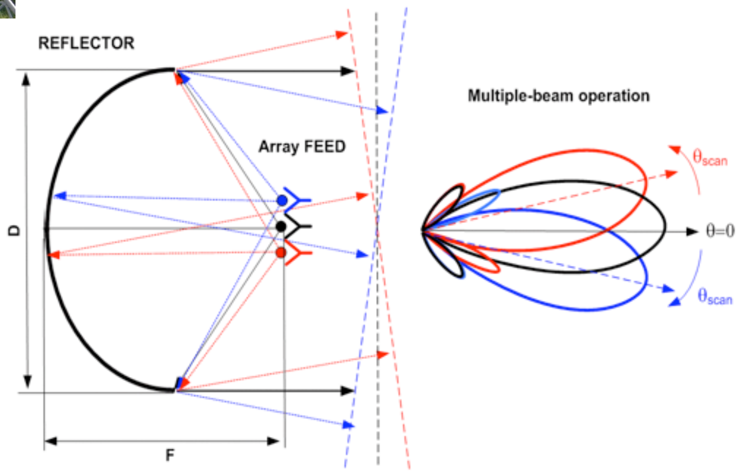
8x8 FPA in WSRT prime focus

APERTIF = APERture Tile In Focus

Increasing the surveying speed by a factor ~ 5 - 25 (depends on T_{sys})







antenna performance

Aperture Efficiency

$A_0 = \eta A$, $\eta = \eta_{sf} \times \eta_{bl} \times \eta_s \times \eta_t \times \eta_{misc}$

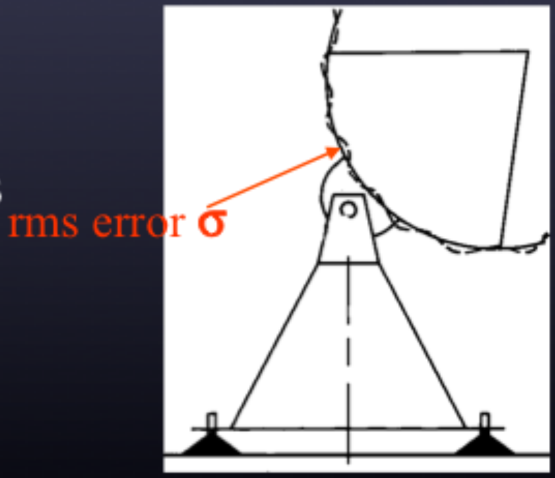
η_{sf} = reflector surface efficiency

η_{bl} = blockage efficiency

η_s = feed spillover efficiency

η_t = feed illumination efficiency

η_{misc} = diffraction, phase, match, loss



$\eta_{sf} = \exp(-(4\pi\sigma/\lambda)^2)$

e.g., $\sigma = \lambda/16$, $\eta_{sf} = 0.5$

importance of antenna element within an interferometer

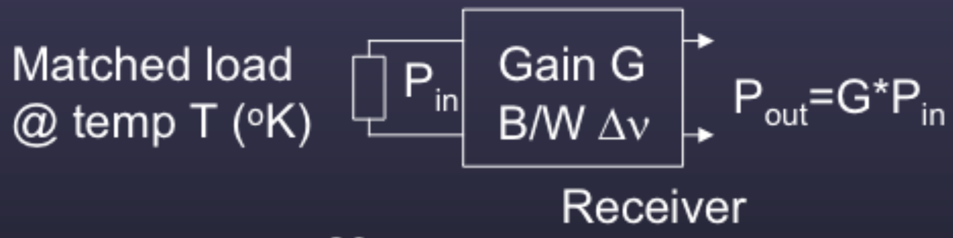
- Antenna amplitude pattern causes amplitude to vary across the source.
- Antenna phase pattern causes phase to vary across the source.
- Polarisation properties of the antenna modify the apparent polarisation of the source.
- Antenna pointing errors can cause time varying amplitude and phase errors.
- Variation in noise pickup from the ground can cause time variable amplitude errors.
- Deformations of the antenna surface can cause amplitude and phase errors, especially at short wavelengths.

observing

Reference received power to the equivalent temperature of a matched load at the input to the receiver

Rayleigh-Jeans approximation to Planck radiation law for a blackbody

$$P_{in} = k_B T \, \Delta\nu \quad (W)$$



k_B = Boltzman's constant (1.38×10^{-23} J/°K)

When observing a radio source, $T_{total} = T_A + T_{sys}$

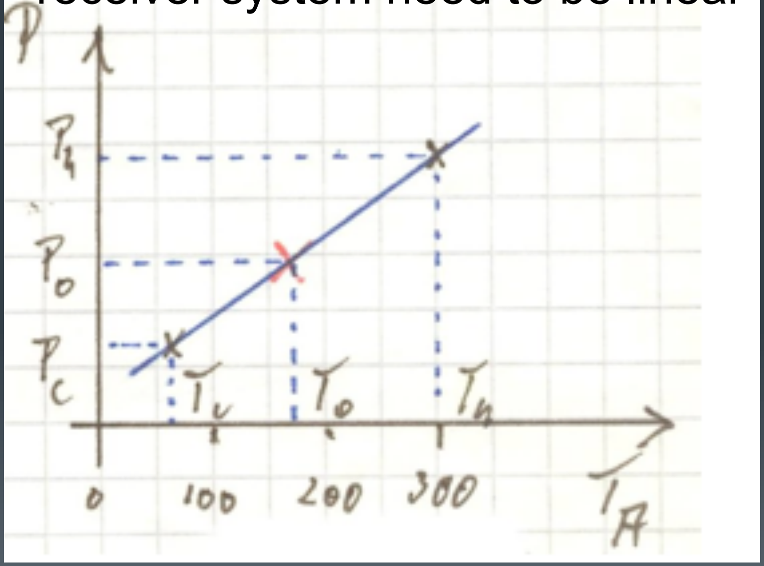
- Tsys = system noise when not looking at a discrete radio source
- T_A = source antenna temperature

calibrate basic step 1

relate the voltages measured at the receiver system to the antenna temperature

$$S_\nu = \frac{2k}{A_{eff}} T_A$$

hot = absorbing material (300 K)
cold = soaked in liquid nitrogen (77 K)



receiver system need to be linear

problem is that we do not know A_{eff} in general
for a horn antenna A_{eff} can be calculated analytical
now we can relate source flux density with antenna temperature

calibrate basic step 2

know flux density of the source can be use to calibrate other telescope

40 Jy

$$S_\nu = \frac{2k}{A_{eff}} T_A$$

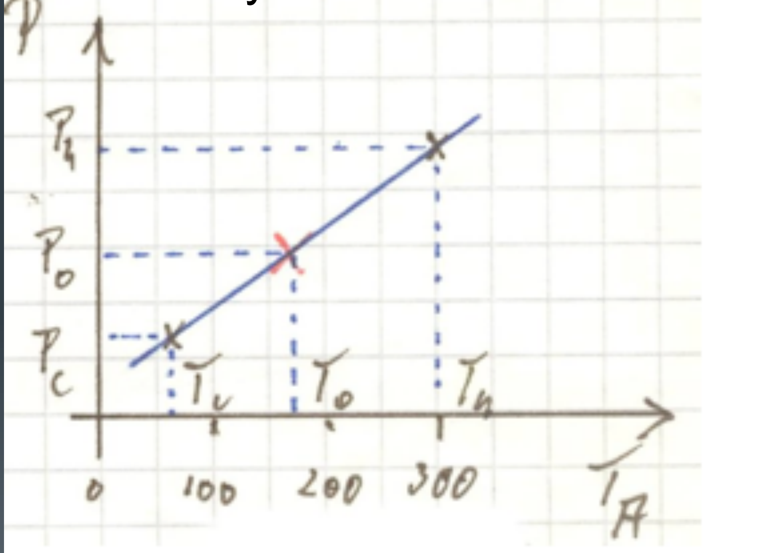
hot = absorbing material (300 K)
cold = soaked in liquid nitrogen (77 K)

antenna temperature for another telescope

$$A_{eff} = 2\,k \cdot \frac{T_{A_0}}{S_0}$$

$$\eta_A = \frac{A_{eff}}{A_{geo}} = \frac{8\,k\,T_{A_0}}{\pi\,D^2\,S_0}$$

receiver system need to be linear



calibrate routine work

with the known parameters of a telescope we can simply bootstrap the flux densities of sources to be measured. All we need is a calibration source not too far away from the target source

$$S_{tgt} = \frac{S_{cal}}{U_{cal}} \cdot U_{tgt}$$

target voltage

calibrator voltage
and flux density

sensitivity (noise)

Unfortunately, the telescope system itself contributes noise to the the signal detected by the telescope, i.e.,

$$P_{out} = P_A + P_{sys} \rightarrow T_{out} = T_A + T_{sys}$$

The *system temperature*, T_{sys}, represents noise added by the system:

$$T_{sys} = T_{bg} + T_{sky} + T_{spill} + T_{loss} + T_{cal} + T_{rx}$$

T_{bg} = microwave and galactic background (3K, except below 1GHz)
T_{sky} = atmospheric emission (increases with frequency--dominant in mm)
T_{spill} = ground radiation (via sidelobes) (telescope design)
T_{loss} = losses in the feed and signal transmission system (design)
T_{cal} = injected calibrator signal (usually small)
T_{rx} = receiver system (often dominates at cm — a design challenge)

Note that T_{bg}, T_{sky}, and T_{spill} vary with sky position and T_{sky} is time variable

radiometer equation

Q: How can you detect T_A (signal) in the presence of T_{sys} (noise)?
A: The signal is correlated from one sample to the next but the noise is not

For bandwidth $\Delta\nu$, samples taken less than $\Delta\tau = 1/\Delta\nu$ are not independent (Nyquist sampling theorem!)

Time τ contains $N = \tau/\Delta\tau = \tau\,\Delta\nu$ independent samples

For Gaussian noise, total error for N samples is $1/\sqrt{N}$ that of single sample

$$\therefore \quad \frac{\Delta T_A}{T_{sys}} = \frac{1}{\sqrt{\tau\,\Delta\nu}}$$

Radiometer equation

$$SNR = \frac{T_A}{\Delta T_A} = \frac{T_A}{T_{sys}}\sqrt{\tau\,\Delta\nu}$$

nice example

When Penzias & Wilson (see lecture 1) made their measurements, they found:

T_{atm} = 2.3 +/- 0.3 K,

T_{loss} = 0.9 +/- 0.4 K,

T_{spill} < 0.1 K.

And they expected T_{sky} ~ 0.

So looking straight up, they expected to measure $T_{A,}$

T_A = 2.3 + 0.9 + 0.1 + 0 = 3.2 K.

What they found was T_A = 6.7 Kelvin!

The excess was the CMB and Galactic emission.

Bell lab advert (right) - 1963 - 3 years before the CMB was detected - and featuring the Penzias & Wilsons horn antenna.