

(1)

Compton- γ parameter and the Sunyaev-Zeldovich effect

Consider electrons and photons co-habitating in a finite medium. In the non-relativistic regime

$$kT_e \ll m_e c^2 \quad , \quad E \ll m_e c^2$$

Obviously in $kT_e \gg E$ Compton scattering will transfer energy to the photons and if $kT_e \ll E$ the photons will heat the electrons. What is in general the fractional energy change of a photon, $\frac{\Delta E}{E}$?

$$\text{Since } E \ll m_e c^2, kT_e \ll m_e c^2 \Rightarrow \frac{\Delta E}{E} = \alpha \frac{E}{m_e c^2} + \beta \frac{kT_e}{m_e c^2}$$

(any second order terms must be negligible)

To find α and β , let's consider two extremes:

Find α : $E \gg kT_e$ (electron practically stationary)

This is Thomson scattering

$$E_f = \frac{E_{in}}{1 + \frac{E_{in}}{m_e c^2} (1 - \cos\theta)} \approx E_{in} \left(1 - \frac{E_{in}}{m_e c^2} (1 - \cos\theta)\right) \Rightarrow$$

$$\frac{E_f - E_i}{E_i} = \frac{\Delta E}{E} = - \frac{E}{m_e c^2} (1 - \cos\theta) \stackrel{\text{average over } \theta}{\Rightarrow} \frac{\Delta E}{E} = - \frac{E}{m_e c^2}$$

\uparrow

$\alpha = -1$

(2)

Find b: $E \ll kT_e$ (Inverse Compton scattering)

$$\frac{dE}{dt} = \frac{4}{3} \sigma_T U_{rad} \left(\frac{v}{c}\right)^2 \gamma^2 \approx \frac{4}{3} \sigma_T c U_{rad} \left(\frac{v}{c}\right)^2$$

P

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \approx 1$$

Rate of photon scattering: $\frac{dN}{dt} = \sigma_T N_{phot} C = \sigma_T U_{rad} \frac{C}{E} C$

Average energy gain per Inverse Compton scattering

$$\Delta E = \frac{\frac{dE}{dt}}{\frac{dN}{dt}} = \frac{\frac{4}{3} \sigma_T c U_{rad}}{\sigma_T U_{rad}/E} \left(\frac{v}{c}\right)^2 \Rightarrow \frac{\Delta E}{G} = \frac{4}{3} \left(\frac{v}{c}\right)^2$$

$$\text{Using } \frac{1}{2} m v^2 = \frac{3}{2} kT \Rightarrow v^2 = \frac{3kT}{m_e} \quad \frac{\Delta E}{G} = 4 \frac{kT}{m_e c^2}$$

S

S₆

$$\frac{\Delta E}{G} = -\frac{e}{m_e c^2} + 4 \frac{kT}{m_e c^2}$$

For $E = 4kT$ no energy transfer

The SZ effect : $kT_e \gg \epsilon$

Energy is transferred to the photons:

$$\frac{\Delta \epsilon}{\epsilon} = \frac{4 k T_e}{m_e c^2}$$

If the region has size l and electron density N_e , the mean free path of an electron due to Thomson scattering is $\lambda_e = \frac{l}{N_e G_T}$

and the optical depth is $\tau_e = N_e G_T l = \frac{l}{\lambda_e}$

For $T_e \gg 1$ the photons undergo a random walk of step λ_e . After how many scatterings N do they escape?

The rms distance a photon travels is $x = N^{1/2} \lambda_e$

Proof by induction in 1D:

$$x_n^2 = N \lambda_e^2 \quad x_{n+1}^2 = \frac{1}{2} (N^{1/2} + 1)^2 \lambda_e^2 + \frac{1}{2} (N^{1/2} - 1)^2 \lambda_e^2$$

$$x_{n+1}^2 = \frac{1}{2} (N + 1 + 2N^{1/2}) \lambda_e^2 + \frac{1}{2} (N + 1 - 2N^{1/2}) \lambda_e^2$$

$$x_{n+1}^2 = (N+1) \lambda_e^2 \Rightarrow x_{n+1} = (N+1)^{1/2} \lambda_e$$

To escape a region of size λ it takes N

$$N = \frac{\lambda^2}{\lambda_e^2} = \tau_e^2$$

For $\tau_e \ll 1$ the number of scatterings is $N = \tau_e^{-2}$

(4)

The Compton γ parameter: $\gamma = \frac{KTe}{mc^2} \max(T_e, Z_e^2)$

Since $\frac{\Delta E}{E} = \frac{4KTe}{mc^2}$, the condition for "significant" energy change before escaping is $\gamma \geq \frac{1}{4}$

Question: With what final energy will a photon escape?

$$\text{After 1 scattering } \frac{e'}{e} = 1 + \frac{4KTe}{mc^2}$$

$$\text{After } N \text{ scatterings } \frac{e'}{e} = \left(1 + \frac{4KTe}{mc^2}\right)^N$$

$$\text{But, because } 4KTe \ll mc^2 \Rightarrow 1 + \frac{4KTe}{mc^2} \approx \exp\left(\frac{4KTe}{mc^2}\right)$$

$$\text{So } \frac{e'}{e} = \exp\left(\frac{4KTe}{mc^2} N\right) = \underline{\exp(4\gamma)} \quad \forall \gamma$$

The S-Z effect: The clusters of galaxies

contain hot gas at temperature $T_e \sim 10^{7.8} \text{ K}$ and are permeated by the cosmic microwave background radiation at $T = 2.73 \text{ K}$ (for redshift $z=0$)

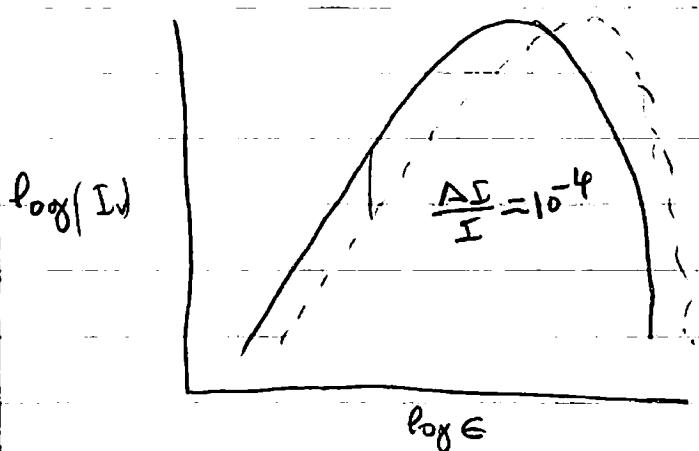
Inverse Compton scattering will increase the energy of photons by $\frac{4KTe}{mc^2}$ per scattering, without changing

the number of photons. In the RJ part of the spectrum this translates to a decrease in brightness temperature:

(5)

In the RJ part of the spectrum

$$\frac{\Delta T}{T} = -2y = -\frac{1}{2} \int \frac{kTe}{mc^2} n_e \sigma_T dl$$



Note: the observed decrement is independent of redshift z , because the scattering results only in a fractional change of the T_{CMB} .

How to measure the Hubble constant with SZ and X-ray observations of clusters

From SZ one obtains $n_e T_e l$

Brennstrahlung gives T_e (from the spectral cutoff) and $L = l^3 n_e^2 T_e^{1/2}$ (from the normalization of the spectrum).

Solve for l , measure the angular size θ and obtain the distance $d_A = \frac{l}{\theta}$

Get z from cluster members lines and obtain H_0 from the angular distance - z relation

$$\text{For } z \ll 1 \quad d_A = \frac{cz}{H_0}$$

(6)

Simple example:

Consider a cluster with plasma density profile:

$$n(r) = \frac{n_c}{1 + \frac{r^2}{r_c^2}}$$

The optical depth is $\tau_0 = \epsilon_T \int_{-\infty}^{\infty} \frac{n_c}{1 + \frac{r^2}{r_c^2}} dr =$

* $= 6.4 \times 10^{-3} n_c f_{\text{XPC}}$ (recall $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \arctan(x) \Big|_{-\infty}^{\infty} = \pi$)

The brak. emissivity is $\epsilon_{SS} \approx 1.4 \times 10^{-22} T^{1/2} n^2 \frac{\text{erg}}{\text{cm}^3 \text{s}}$

The x-ray luminosity is

* $L_x = 4\pi \int_0^{r_c} \epsilon_{SS} r^2 dr = 2.5 \times 10^{41} s^{-1} n_c^2 r_{c,\text{kpc}} \left(\frac{K T_e}{\text{keV}} \right)^{1/2} \frac{\text{erg}}{\text{s}}$

f: ratio of the x-ray emitting volume to the core volume of the cluster.

Typical values: $L_x = 10^{44} h^{-2}$ $r_c = 200 h^{-1} \text{kpc}$ $T_e \approx 4 \text{ keV}$

These give from * $n_c = 5 \times 10^{-3} h^{-1} s^{-1/2}$

Using this in * we obtain $\tau_0 = 6.4 \times 10^{-3} (hf)^{-1/2}$

For $\frac{\Delta T}{T} = -10^{-4}$

Then $\frac{\Delta T}{T} = -2 \tau_0 \frac{K T_e}{mc^2} = -10^{-4} (hf)^{1/2}$

Measure $\frac{\Delta T}{T}$, model f to obtain h.