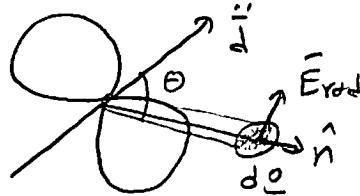


Question 2.

We found in class a minimum value b_{\min} for the impact parameter using the uncertainty principle.

Using energy conservation and the fact that bremsstrahlung emission has a cutoff to find a similar b_{\min} .



(8)

Dipole moment $\bar{d} = q \bar{x}$ $\ddot{\bar{d}} = q \dot{\bar{v}}$

So the Larmor formula becomes $P = \frac{2 \ddot{\bar{d}}^2}{3c^3}$

How can we find the spectrum of the emitted radiation?

FT of dipole moment d :

$$d(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \bar{d}(\omega) d\omega \Rightarrow \ddot{\bar{d}}(t) = - \int_{-\infty}^{\infty} \omega^2 e^{-i\omega t} \bar{d}(\omega) d\omega$$

$$\text{So } E_{rad}(t) = \frac{\ddot{\bar{d}}(t)}{Rc^2} \sin \theta = - \int_{-\infty}^{\infty} \omega^2 e^{-i\omega t} \bar{d}(\omega) \frac{\sin \theta}{Rc^2} d\omega$$

$$\text{But } E_{rad}(t) = \int_{-\infty}^{\infty} e^{-i\omega t} \bar{E}_{rad}(\omega) d\omega$$

$$\text{So, } \bar{E}_{rad}(\omega) = - \omega^2 \bar{d}(\omega) \frac{\sin \theta}{Rc^2}$$

$$\text{Now } \frac{dW}{dA2\epsilon} = |S| = \frac{c}{4\pi} E_{rad}^2(t) \Rightarrow \frac{dW}{dA} = \frac{c}{4\pi} \int_{-\infty}^{\infty} E_{rad}^2(t) dt$$

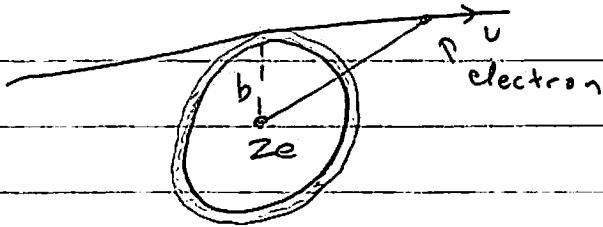
$$= c \int_0^{\infty} |\bar{E}_{rad}(\omega)|^2 d\omega = \frac{1}{Rc^3} \int_0^{\infty} \omega^4 |\bar{d}(\omega)|^2 \sin^2 \theta d\omega$$

$$\text{or } \frac{dW}{d\omega d\Omega} = \frac{\omega^4}{c^3} |\bar{d}(\omega)|^2 \sin^2 \theta d\omega$$

$$\Rightarrow \boxed{\frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} |\bar{d}(\omega)|^2}$$

9

THERMAL BREMSS TRAHLUNG



$$\bar{J}(\omega) = -\frac{1}{2\pi\omega^2} \int_{-\infty}^{\infty} e^{i\omega t} \bar{j}(t) dt = -\frac{e}{2\pi\omega^2} \int_{-\infty}^{\infty} e^{i\omega t} \bar{v}(t) dt$$

Characteristic collision time? $\tau \sim \frac{b}{v}$ } better limit than ∞

What is the maximum ω for which we expect emission?

$$\omega \sim \frac{1}{\tau} \sim \frac{v}{b}$$

For $\omega > \frac{1}{\tau}$ the integrand oscillates rapidly and $\bar{J}(\omega) \approx 0$.

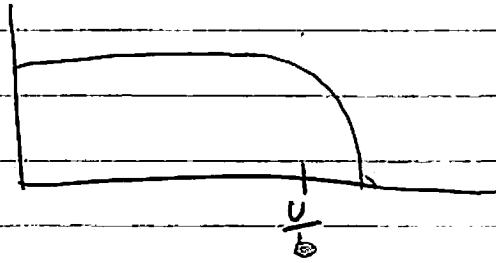
For $\omega \ll \frac{1}{\tau}$, $e^{i\omega t} \approx 1$ and

$$\bar{J}(\omega) \approx \frac{e}{2\pi\omega^2} \Delta v$$

$$\text{So } \frac{dW}{d\omega} = \frac{8\pi\omega^4}{3c^3} |\bar{J}(\omega)|^2 = \frac{8\pi\omega^4}{3c^3} \frac{e^2}{4\pi\omega^4} \Delta v^2 = \underline{\frac{2e^2}{3\pi c^3} |\Delta v|^2}$$

Note: no frequency dependence!

The emission is flat up to $\omega \sim \frac{v}{b}$ and then it dies!



What is $|\Delta v|$? $|\Delta v| \approx |\alpha| \tau$

The only acceleration that contributes to the time averaged emission is $\perp \vec{v}$. The $\parallel \vec{v}$ component flips sign at the closest approach and cancels out

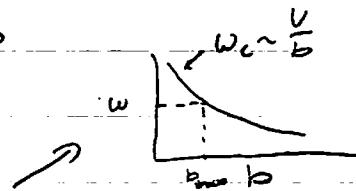
$$\alpha \approx \frac{ze^2}{m_e b^2}, \tau \sim \frac{b}{v} \Rightarrow |\Delta v| \approx \frac{ze^2}{m_e b v} \quad \text{(here we miss a factor of 2 from the exact calculation)}$$

$$\text{So } \frac{dW(b)}{dw} = \frac{2e^2}{3\pi c^3} \frac{z^2 e^4}{m_e^2 b^2 v^2}$$

number of e passing through an annulus of width db at b
per unit time $= n_e v 2\pi b db$

FOR MONO-
ENERGETIC
ELECTRONS

$$E_{ff, \text{mono}}(w) \approx n_i n_e 2\pi v \int_{b_{\min}}^{b_{\max}} \frac{dW(b)}{dw} b db$$



$b_{\max} \sim \frac{v}{w}$ because for larger b_{\max} $\frac{v}{b} \sim \frac{1}{\tau} < w$
∴ we cannot produce such a high frequency

$\Delta x \Delta p \geq \hbar$ $b_{\min} m_e v \geq \hbar \Rightarrow b_{\min} \sim \frac{\hbar}{m_e v}$, smaller b_{\min} does not make sense

$$E_{ff, \text{mono}}(w) = \frac{16 n e^6}{3 \sqrt{3} c^3 m_e^2 v} n_i n_e z^2 g_{ff}(v, w)$$

Gauant factor $g_{ff}(v, w) = \frac{\sqrt{3}}{\pi} \ln \left(\frac{b_{\max}}{b_{\min}} \right)$

(11)

Integrating over a Maxwell-Boltzmann distribution
 $\propto v^2 e^{-\left(\frac{mv^2}{2kT}\right)}$ we obtain

$$\epsilon_{ff}(v) = 6.8 \cdot 10^{-38} n_e n_i z^2 T^{-1/2} e^{-\frac{hv}{kT}} \bar{g}_{ff} \text{ CGS}$$

$$\bar{g}_{ff} \sim 1.5$$

velocity averaged
Gauß factor

Integrating over frequency
we obtain:

$$\epsilon_{ff} = \frac{dW}{dt dv} = 1.4 \cdot 10^{-27} T^{1/2} n_e n_i z^2 \bar{g}_B(T)$$

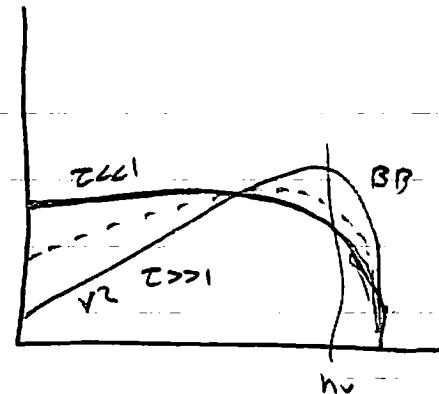
$$\bar{g}_B(T) = 1.1 - 1.5 \quad 1.2 \\ 15 \text{ 20\%} \\ \text{OK}$$

From Bremsstrahlung to BB

optical depth of a source to
its Bremsstrahlung photons:

= optical depth to Compton

scattering = $\tau = n_e \sigma R \ll \text{source size}$



Photons are up-scattered to energies close to $kT = h\nu$

For $\tau >> 1$ we obtain a BB

$$B_v(T) = \frac{2h\nu^3/c^2}{e^{\frac{h\nu}{kT}} - 1}$$