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General relativistic model for experimental measurement of the speed of propagation of gravity by VLBI

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Abstract. A relativistic sub-picosecond model of gravitational time delay in radio astronomical observations is worked out and a new experimental test of general relativity is discussed in which the effect of retardation of gravity associated with its finite speed can be observed. As a consequence, the speed of gravity can be measured by differential VLBI observations. Retardation in propagation of gravity is a central part of the Einstein theory of general relativity which has not been tested directly so far. The idea of the proposed gravitational experiment is based on the fact that gravity in general relativity propagates with finite speed so that the deflection of light caused by the body must be sensitive to the ratio of the body's velocity to the speed of gravity. The interferometric experiment can be performed, for example, during the very close angular passage of a quasar by Jupiter. Due to the finite speed of gravity and orbital motion of Jupiter, the variation in its gravitational field reaches observer on Earth not instantaneously but at the retarded instant of time and should appear as a velocity-dependent excess time delay in addition to the well-known Shapiro delay, caused by the static part of the Jupiter's gravitational field. Such Jupiter-QSO encounter events take place once in a decade. The next such event will occur on September 8, 2002 when Jupiter will pass by quasar J0842+1835 at the angular distance $3.7'$. If radio interferometric measurement of the quasar coordinates in the sky are done with the precision of a few picoseconds ($\sim 5\ \mu$as) the effect of retardation of gravity and its speed of propagation may be measured with an accuracy about 10%.

1. Theoretical Background

Experimental verifications of the basic principles underlying Einstein's general relativity theory are important for fundamental physics. All previous experimental tests of general relativity in the solar system have relied upon the static Schwarzschild solution (Will 1993) and, therefore, were not sensitive to the effects entirely associated with the propagation speed of gravity. It is worth noting that gravitational waves are inherent to the radiative (far) zone of a system emitting the waves (Misner, Thorne & Wheeler 1973; Barish & Weiss 1999). However, the gravitational waves do not propagate freely through the interior of a non-radiative (near) zone of the system. Nevertheless, the process of generation of gravitational waves produces retarded effects in the near zone leading to appearance of the gravitational radiation reaction force in the relativistic equations of motion of extended bodies comprising a self-gravitating astronomical system (Damour et al. 1989). Existence of this force is a consequence of the finite speed of propagation of gravity as it was experimentally confirmed by Taylor (1994).

We have found (Kopeikin 2001) that the gravitational bending of light passing through the gravitational field of a moving massive object like Jupiter, though being dominated by the spherically-symmetric component of its gravitational field, also contains terms associated with the finite speed of propagation of gravity. Our calculations reveal that electromagnetic signals interact with the light-ray deflecting bodies only through the retarded gravitational fields – the observational effect which must be accounted for in precise data processing algorithms adopted for the microarcsecond space astrometry.

This gravitational light-propagation theory, in the case of a static spherically-symmetric field, gives the same result as that predicted by Einstein for the bending of light, $\alpha_E \simeq 4GM/(c^2R\theta)$, where M is the mass of the light-ray deflecting body, R is the distance from observer to the body, and θ is the (small) angle in the sky between the undisturbed geometric positions of the source of light and the center of mass of the massive body. Furthermore, this theory allows to calculate the correction, α_{PG}, to the Einstein deflection α_E related to the variability of the gravitational field produced by the motion of the light-ray deflecting body. We were successful in proving (Kopeikin 2001) that these corrections in the bending of light are inherently associated with the finite speed of propagation of gravity and in case of slowly moving bodies can be parameterized as $\alpha_{PG} \simeq (1+\delta)(\alpha_E/\theta)(v/c)$, where v is the orbital velocity of the light-ray deflecting body with respect to the barycenter of the solar system projected on the plane of the sky, and $\delta = c_g/c - 1$ is a fitting parameter used in data analysis. It is chosen such that $\delta = 0$, if the speed of gravity c_g equals the speed of light c.

Parameter δ is a close analogue of the parameter $\alpha_2 = (c_g/c)^2 - 1$ of the parameterized post-newtonian (PPN) formalism which quantifies possible violation of the local Lorentz invariance (Will 1993)[1]. It was shown (Nordtvedt 1987) that $\alpha_2 < 4 \times 10^{-7}$ under the (rather restrictive) assumption that the preferred frame is realized by the cosmological Hubble flow. If one abandons any anthropic assumption about the speed of the solar system with respect to an (actually unknown) preferred frame the limit on $\alpha_2 < 0.1$ can be obtained from the analysis of the anomalous perihelion shifts of inner planets (Will 1993). The primary purpose of our experiment is, however, to observe directly the effect of retardation in propagation of gravitational field in the solar system rather than improving limits on α_2. Nevertheless, we would like to emphasize that relativistic effects in propagation of light through time dependent gravitational fields are also sensitive to violation of local Lorentz invariance.

The largest measurable contribution to the variable, time-dependent part of the solar system gravitational field comes out from the orbital motion of Jupiter. The minimal value of the impact parameter of an incoming light ray from a quasar that can be achieved for Jupiter is also the least possible amongst all the solar system bodies. Therefore, it is highly sensible to make an attempt for detection of the "gravity retardation" effect by observing very accurately the gravitational deflection of light from a background source (quasar) caused by the motion of Jupiter around the barycenter of the solar system. As explained in (Kopeikin 2001) the magnitude of the observed effect is directly translated to the measured value of the propagation speed of gravity c_g. This is the essence of the new test of general relativity which has never been done before with sufficient accuracy.

Radio astronomical methods of VLBI are the most accurate for measuring gravitational deflection of electromagnetic waves. The two most precise measurements of the bending of radio waves near the sun (Lebach et al. 1995; Robertson, Carter & Dillinger 1991) were accurate to about 0.1%. However, these observations were insensitive to the speed of gravity effect α_{PG} because of the relatively large impact parameter of the incoming light ray. Even at the solar limb, the magnitude of α_{PG} is $\sim 10^{-5}$ of the static gravitational bending $\alpha_E = 1.75'$ and is totally unobservable because of the highly turbulent solar magnetosphere.

On September 8, 2002 Jupiter will pass at an angular distance of $3.7'$ from the quasar J0842+1835 making an ideal celestial configuration for measuring the speed of propagation of gravity by using the phase-referencing VLBI technique (Fomalont & Kopeikin 2002). The encounter in 2002 is especially favorable because: (1) it occurs when Jupiter is relatively far from the Sun (the next near occultation which occurs is a few degrees from the Sun), and (2) the five critical hours of the closest approach occur when Jupiter is near the transit line for VLBA observations.

We have estimated that for this Jupiter-quasar encounter the deflection from the static gravitational field and from the propagation of gravity are, respectively, $\alpha_E = 1.26$ mas and $\alpha_{PG} = 53$ μas [2] both in the plane of the sky with the static bending radially from Jupiter and the propagation bending in the direction of Jupiter's motion (see Eqs. (7) and (8) in Sec. 3). As one can see the ratio $|\alpha_{PG}|/\alpha_E \simeq 0.04$ is much larger for Jupiter than for the Sun which is explained by the ability to get a smaller impact parameter θ for the light ray passing by Jupiter than that for the Sun.

2. Relativistic Model of VLBI Time Delay

Detection of the effect of gravity propagation requires a more advanced VLBI model for light propagating in the time dependent gravitational field of the solar system. Such a model must be valid to a precision of better than 0.1 ps. In the present paper we discuss the appropriate corrections to the standard Shapiro time delay (Shapiro 1967) which bring the accuracy of the model up to the necessary threshold.

The general formula for the relativistic time delay ΔT in the field of a system of moving bodies is given in (Kopeikin 2001)

$$\Delta T = (1+\gamma)\frac{G}{c^3}\sum_{a=1}^{N} m_a \int_{s_0}^{s} \frac{\left(1-\frac{1}{c}\boldsymbol{k}\cdot\boldsymbol{v}_a(\zeta)\right)^2 \Upsilon(\zeta)\, d\zeta}{t^* - \zeta + \frac{1}{c}\boldsymbol{k}\cdot\boldsymbol{x}_a(\zeta)}, \quad (1)$$

where $\Upsilon(\zeta) = 1/\sqrt{1-c^{-2}v_a^2(\zeta)}$ is the Lorentz factor, γ is the PPN parameter (Will 1993), m_a is the mass of the ath body, t^* is the time of the closest approach of electromagnetic signal to the barycenter of the Solar system [3], $\boldsymbol{x}_a(t)$ are coordinates of the ath body, $\boldsymbol{v}_a(t) = d\boldsymbol{x}_a(t)/dt$ is the (non-constant) velocity of the ath light-ray deflecting body, $\boldsymbol{k}$ is the unit vector from the point of emission to the point of observation, s is a retarded time obtained by solving the gravitational null cone equation for the time of observation of photon $t = s + c_g^{-1}|\boldsymbol{x} - \boldsymbol{x}_a(s)|$, and s_0 is found by solving the same equation written down for the time of emission of the photon $t_0 = s_0 + c_g^{-1}|\boldsymbol{x}_0 - \boldsymbol{x}_a(s_0)|$.

One emphasizes that the equations for the retarded times depend on the speed of gravity c_g, but not the speed of light c. This is because they were obtained by solving Einstein equations for the space-time metric perturbations by making use of retarded Lienard-Wiechert tensor potentials (Kopeikin & Schäfer 1999). These retarded gravitational potentials describe propagation of gravity without any relation to the problem of propagation of light in the gravitational field. In general relativity $c_g = c$ numerically. However, when light propagates through time-dependent gravitation field, in principle, one can separate relativistic effects associated with propagation of light and gravity.

The Earth and Sun also contribute significantly to the gravitational time delay and must be included in the data

[1] One notices that $\delta = \alpha_2/2$ in the first approximation.

[2] It corresponds to the time delays 122.2 and 5.1 picoseconds respectively on a baseline $b = 6000$ km.

[3] The time t^* is used in calculations as a mathematical tool only. It has no real physical meaning because of its dependence on the choice of a coordinate system.

processing algorithm in order to extract accurately the effect of retardation of gravity. Precise calculation of the integral (1) for two radio antennas gives the differential VLBI time delay

$$\Delta \ = \ \Delta_2 T - \Delta_1 T = \Delta_\oplus + \Delta_\odot + \Delta_J + \Delta_{JPG} \ . \tag{2}$$

The first term in the right hand side of (2) describes the gravitational (Shapiro) time delay due to the gravitational field of the Earth

$$\Delta_\oplus \ = \ (1+\gamma)\frac{GM_\oplus}{c^3}\ln\frac{X_1 + \boldsymbol{K}\cdot\boldsymbol{X}_1}{X_2 + \boldsymbol{K}\cdot\boldsymbol{X}_2} \ . \tag{3}$$

It can reach 21 ps for the baseline $b = 6000$ km.

The second term in the right hand side of (2) describes the gravitational (Shapiro) time delay due to the Sun

$$\Delta_\odot \ = \ (1+\gamma)\frac{GM_\odot}{c^3}\ln\frac{r_{1\odot} + \boldsymbol{K}\cdot\boldsymbol{r}_{1\odot}}{r_{2\odot} + \boldsymbol{K}\cdot\boldsymbol{r}_{2\odot}} \ . \tag{4}$$

It can vary (for $b = 6000$ km) from 17×10^4 ps for the light ray grazing the Sun's limb to only 17 ps when direction to the source of light is opposite to the Sun.

The third term in the right hand side of (2) is the Shapiro time delay due to the static part of the gravitational field of Jupiter

$$\Delta_J \ = \ (1+\gamma)\frac{GM_J}{c^3}\left(1 + \boldsymbol{K}\cdot\boldsymbol{v}_J\right)\ln\frac{r_{1J} + \boldsymbol{K}\cdot\boldsymbol{r}_{1J}}{r_{2J} + \boldsymbol{K}\cdot\boldsymbol{r}_{2J}} \ . \tag{5}$$

Finally, the forth term in the right hand side of (2) is the time delay caused by the finite speed of gravity as predicted by general relativity theory (Kopeikin 2001)

$$\Delta_{JPG} \ = \ 2(1+\delta)\frac{GM_J}{c^4}\frac{\boldsymbol{b}\cdot\boldsymbol{v}_J + (\boldsymbol{b}\cdot\boldsymbol{N}_{1J})(\boldsymbol{K}\cdot\boldsymbol{v}_J)}{r_{1J} + \boldsymbol{K}\cdot\boldsymbol{r}_{1J}} \ . \tag{6}$$

In formulas (3)–(6) we use the following notations: $M_\oplus$ – mass of the Earth, $M_\odot$ – mass of the Sun, M_J – mass of Jupiter, $\boldsymbol{v}_J(t_1)$ – the barycentric velocity of Jupiter, and $\boldsymbol{K}$ – the unit vector from the barycenter of the solar system to the quasar observed. Also, for each $i = 1, 2$, one has the baseline vector $\boldsymbol{b} = \boldsymbol{X}_1 - \boldsymbol{X}_2$, $r_{i\odot} = |\boldsymbol{r}_{i\odot}|$, $r_{iJ} = |\boldsymbol{r}_{iJ}|$, $\boldsymbol{N}_{1J} = \boldsymbol{r}_{1J}/r_{1J}$, $\boldsymbol{r}_{i\odot} = \boldsymbol{x}_i(t_i) - \boldsymbol{x}_\odot(t_i)$, $\boldsymbol{r}_{iJ} = \boldsymbol{x}_i(t_i) - \boldsymbol{x}_J(t_i)$, $\boldsymbol{x}_i = \boldsymbol{x}_\oplus(t_i) + \boldsymbol{X}_i(t_i)$, where $\boldsymbol{X}_i(t_i)$ are the geocentric coordinates of i-th VLBI station, $\boldsymbol{x}_\oplus$ – the barycentric coordinates of the geocenter, $\boldsymbol{x}_\odot$ – barycentric coordinates of the Sun, $\boldsymbol{x}_J$ – barycentric coordinates of Jupiter, and t_i is time of arrival of the plane front of electromagnetic wave from quasar to the ith VLBI station.

We notice there are two relativistic parameters to be measured in order to test validity of general relativity theory - the PPN parameter γ, and the speed of gravity parameter $\delta = c_g/c - 1$. The best experimental measurement of parameter γ had been conducted by Lebach et al. (1995) who obtained $\gamma = 0.9996 \pm 0.0017$ in an excellent agreement with general relativity. The primary goal of the new experimental test of general relativity is to measure the parameter δ which will set up limits on the numerical value of the speed of gravity c_g (Fomalont & Kopeikin 2002).

During the passage of Jupiter near the quasar the time-dependent impact parameter $\boldsymbol{\xi}(t)$ of the light ray with respect to Jupiter will be always small as compared with the distance from Earth to Jupiter which will be approximately 6 AU. It is convenient to introduce the unit vector $\mathbf{n} = \boldsymbol{\xi}/|\boldsymbol{\xi}|$ along the direction of the impact parameter according to definition $\sin\theta\ \mathbf{n} = (\mathbf{K}\times(\mathbf{N}_{1J}\times\mathbf{K}))$, where θ is a small angle between the unperturbed astrometric position of the quasar and that of Jupiter. Making use of the (impact parameter θ) expansion $\mathbf{N}_{1J} = -(1-\theta^2/2)\mathbf{K}+\theta\,\mathbf{n}+O(\theta^3)$, we obtain the functional structure of the Shapiro time delay Δ_J and the speed of gravity delay Δ_{JPG} in a more explicit form (assuming for simplicity $\gamma = 1$)

$$\Delta_J \ = \ \frac{4GM_J}{c^3 r_{1J}}\left[\frac{\boldsymbol{n}\cdot\boldsymbol{B}}{\theta} + \frac{(\boldsymbol{n}\cdot\boldsymbol{B})^2}{r_{1J}\theta^2} - \frac{(\boldsymbol{K}\times\boldsymbol{B})^2}{2r_{1J}\theta^2}\right] \ , \tag{7}$$

$$\Delta_{JPG} \ = \ (1+\delta)\frac{4GM_J}{c^4 r_{1J}}\frac{\boldsymbol{b}\cdot\boldsymbol{v}_J - (\boldsymbol{K}\cdot\boldsymbol{v}_J)(\boldsymbol{K}\cdot\boldsymbol{b})}{\theta^2} \ , \tag{8}$$

where $\boldsymbol{B} = \boldsymbol{b} - c^{-1}(\boldsymbol{K}\cdot\boldsymbol{b})(\boldsymbol{v}_2 - \boldsymbol{v}_J) + O(c^{-2})$, and all quantities in the right sides of Eqs. (7)–(8) are taken at the time t_1.

3. The Effect of the Magnetosphere of Jupiter

In addition to various special and general relativistic effects in the time of propagation of electromagnetic waves from the quasar to the VLBI antenna network, we must account for the effects produced by the Jovian magnetosphere. Measurements obtained during the occultations of Galileo by Jupiter indicate (Flasar et al. 1997) that near the surface of Jupiter the electron plasma density reaches the peak intensity $N_0 = 1.0\times 10^{10}$ m^{-3}. We shall assume that the Jovian magnetosphere is spherical [4] and a radial drop-off of the plasma density $N(r)$ is proportional to $1/r^{2+A}$ where r is the distance from the center of Jupiter. The guess is that $A \geq 0$, and we will assume that $A = 0$ for the worst possible case. Hence, radial dependence of the electron plasma density is taken as $N(r) = N_0(R_J/r)^{2+A}$, where $R_J = 7.1\times 10^7$ m is the mean radius of Jupiter.

The plasma produces a delay ΔT in the time of propagation of radio signal which is proportional to the column plasma density in the line of sight given by integral (Yakovlev 1989)

$$N_l \ = \ \int_d^{r_0}\frac{N(r)\,dr}{r^{A+1}\sqrt{r^2-d^2}} + \int_d^{r_1}\frac{N(r)\,dr}{r^{A+1}\sqrt{r^2-d^2}} \ , \tag{9}$$

where r_0 and r_1 are radial distances of quasar and radio antenna from Jupiter respectively, and $d = |\boldsymbol{\xi}|$ is the impact parameter of the light ray from the quasar to Jupiter

[4] In reality the magnetosphere has a dipole structure and we speculate that our model which assumes circular symmetry grossly underestimates the plasma content along the polar direction where the closest approach occurs.

[5]. In the experiment under discussion the impact parameter is much less than both r_0 and r_1. Hence,

$$N_l(\mathrm{m}^{-2}) \;=\; N_0 R_J \left(\frac{R_J}{d}\right)^{A+1} \frac{\sqrt{\pi}\Gamma\left(\frac{A+1}{2}\right)}{\Gamma\left(1+\frac{A}{2}\right)} \;, \qquad (10)$$

where $\Gamma(z)$ is the Euler gamma-function. The plasma time delay

$$\Delta T(\mathrm{s}) \;=\; 40.4\, c^{-1}\nu^{-2} N_l \;, \qquad (11)$$

where c is the speed of light in vacuum measured in m/sec, ν is the frequency of electromagnetic signal measured in Hz.

It is worthwhile noting that, in fact, the VLBI array measures difference in path length between the radio telescopes. Hence, one has to differentiate N_l in expression (11) with respect to the impact parameter d and project the result on the plane of the sky. This gives a magnetospheric VLBI time delay of

$$\Delta_{JM}(\mathrm{ps}) \;=\; 6.3\times 10^{-7}(A+1)\frac{N_l}{d}\left(\frac{\nu_0}{\nu}\right)^2 \frac{\boldsymbol{n}\cdot\boldsymbol{b}}{c} \;, \qquad (12)$$

normalized to the frequency $\nu_0 = 8.0$ GHz. Substituting $d = 13R_J$ and taking the baseline $b = 6000$ km we find

$$\Delta_{JM} \;=\; 2.34\,(\nu_0/\nu)^2 \;\; \mathrm{ps} \qquad (A=0)\;, \qquad (13)$$

$$\Delta_{JM} \;=\; 0.13\,(\nu_0/\nu)^2 \;\; \mathrm{ps} \qquad (A=1)\;, \qquad (14)$$

$$\Delta_{JM} \;=\; 0.03\,(\nu_0/\nu)^2 \;\; \mathrm{ps} \qquad (A=2)\;. \qquad (15)$$

This represents the pure bending from Jupiter's magnetosphere which should be compared with the propagation of gravity time delay $\Delta_{JPG} = 5.1$ ps at closest approach.

If we observe at the dual frequencies at 2.3 GHz and 8.4 GHz in the normal geodetic mode, we certainly can determine the ionospheric (both from Jupiter and from the Earth) effects. However, our sensitivity at 8.4 GHz will be decreased because of only one polarization and only half the total bandwidth. The noise in the ionosphere/magnetosphere bending may also be a limit. As a rough order of magnitude, the position error per day is 10 μas at 8.4 GHz and 30 μas at 2.3 GHz. Whatever bending we obtain at 2.3 GHz, about 10% is removed from the 8.4 GHz bending. Thus, the ionosphere/magnetosphere correction will have an error of 3 to 4 μas.

Only in the worst case scenario will the effects of Jupiter's magnetosphere be significant at 8 GHz observing. Perhaps we should observe at two widely spaced frequencies in the 8 GHz band, say at 8.0 GHz and 8.5 GHz, then, independently reduce the data at the two frequencies. The gravitational bending delay (7) and gravitational retardation of gravity delay (8) are both independent of frequency. Any plasma delay should scale inversely with the frequency-squared and can be determined by looking at the difference measurements. However, this small frequency difference will produce very large errors in the estimate of the Jovian bending and also be strongly affected by Earth ionospheric contamination.

If we observe at 15 GHz instead, the magnetospheric and ionospheric delays are both a factor of four smaller and almost certainly negligible. However, the system sensitivity is less and one of the calibrators may be too weak to reliable detect (J0839+1802). We are still unsure about the most optimum method to deal with the possible Jovian magnetosphere component.

Time variability of the Jupiter magnetosphere could cause problems. For example, if there were very large, chaotic changes in the Jovian magnetosphere, then we could lose coherence over a minute of time. However, this fluctuation model is very pessimistic and unlikely, and would probably average out to the steady state model.

4. Summary

We believe that the differential VLBI experiment in September 2002 can measure the retardation effect in propagation of gravity and determine the speed c_g of its propagation with 10% to 20% accuracy. If the experiment is successful it will provide a new independent test of general relativity in the solar system.

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References

Barish B.C. & Weiss R. , Physics Today, **52**, 44 (1999)

Damour, T., Grishchuk, L.P., Kopeikin, S.M. & Schäfer, G., In: Proc. 5th M. Grossman Meeting on Gen. Rel., Eds. D. Blair et al. (Singapore: World Scientific Publisher, 1989), p. 451

Flasar F.M., Hinson D.P., Kliore A.J., Schinder P.J. & Twicken J.D., BAAS, **29**, 1018 (1997)

Fomalont E. & Kopeikin S., *Phase Referencing Using Several Calibrator Sources*, these proceedings

Kopeikin S.M., ApJL, **556**, 1 (2001)

Kopeikin S.M. & Schäfer G., Phys. Rev. D, **60**, 124002 (1999)

Lebach D.E., Corey B.E., Shapiro I.I., Ratner M.I., Webber J.C., Rogers A.E.E., Davis J.L. & Herring T.A., Phys. Rev. Letters, **75**, 1439 (1995)

Misner C.W., Thorne K.S. & Wheeler J. A., Gravitation (Freeman: San-Francisco, 1973)

Nordtvedt, K., ApJ, **320**, 871 (1987)

Robertson D.S., Carter W.E. & Dillinger W.H., Nature, **349**, 768 (1991)

Shapiro I.I., Science, **157**, 806 (1967)

Taylor J.H., Rev. Mod. Phys., **66**, 711 (1994)

Treuhaft R.N. & Lowe S.T., Astron. J (USA), **102**, 1879 (1991)

Will C.M., Theory and Experiment in Gravitational Physics (Cambridge University Press: Cambridge, 1993)

Yakovlev O.I., Radio Wave Propagation in Space (Krieger Publishing Company: Malabar, Florida 1989)

[5] This impact parameter $d \simeq 14R_J$ on September 8, 2002 and corresponds to the angle $\theta = 3.7'$ between the quasar and Jupiter in the plane of the sky.