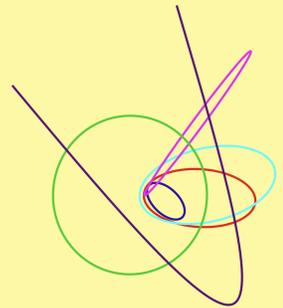


Orbits of Stars in the Central Parsec

A Hybrid N-body Code Incorporating Algorithmic Regularization and Post-Newtonian Forces

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Abstract

We describe a novel N -body code designed for simulations of the central regions of galaxies containing massive black holes. The code incorporates Mikkola's "algorithmic" chain regularization scheme including post-Newtonian terms up to PN2.5 order. Stars moving beyond the chain are advanced using a fourth-order integrator with forces computed on a GRAPE board. Performance tests confirm that the hybrid code achieves better energy conservation, in less elapsed time, than the standard scheme and that it reproduces the orbits of stars tightly bound to the black hole with high precision. The hybrid code is applied to two sample problems: the effect of finite- N gravitational fluctuations on the orbits of the S-stars; and inspiral of an intermediate-mass black hole into the galactic center.

The hybrid N-body code

The basic idea of the hybrid N -body code is depicted in Fig. 1. Orbits of particles close to the central BH, i.e. with r_{crit} (red circle), are precisely integrated in the AR-CHAIN part of the code. This also takes into account perturbations from stars within r_{perturb} (blue circle). Particles outside of r_{perturb} only act upon the center-of-mass motion of the chain. Outside the chain, orbits are integrated using the standard Hermite scheme ϕ GRAPE. The GRAPE hardware is used in this part of the calculation to achieve maximal speed. Again depending on distance, particles may either react to the chain's center of mass or the resolved chain. At the end of every step, checks are performed to find particles that enter or leave the chain, and, in case it is needed, treated accordingly. The number of particles integrated in the chain is typically of order of a few up to a few tens.

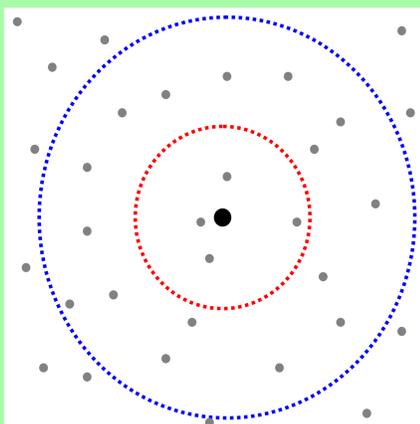


Fig. 1: Schematic view of the hybrid N -body code. Dots represent the central BH (black) and surrounding stars (grey). Stars within r_{crit} (red circle) are treated in the AR-CHAIN taking into account perturbations from stars inside r_{perturb} (blue circle).

Performance tests

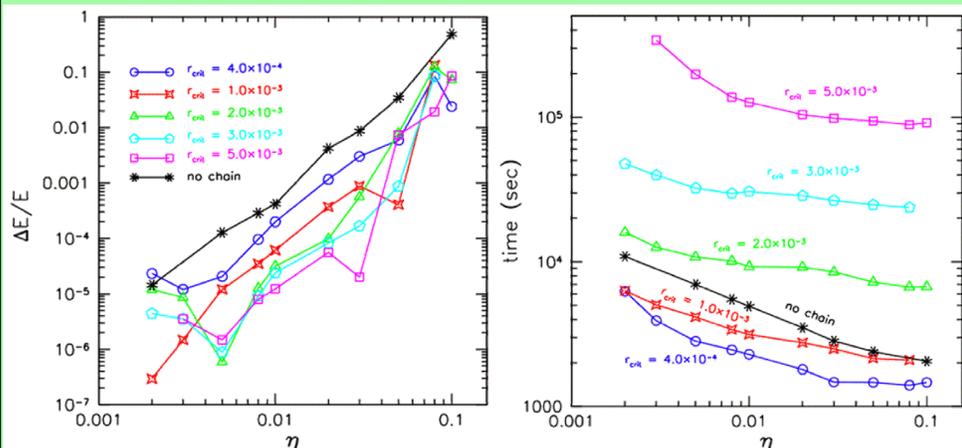


Fig. 2: Results from performance tests: shown on the left is energy conservation in integrations until $t=1$ ($\sim 10^4$ yr) for a model with 10^4 particles. Black line (asterisks) are for ϕ GRAPE without the regularized chain. Elapsed time for the same integrations can be seen on the right.

We tested the performance of the hybrid code using various realizations of a model designed to mimic the density profile of the star cluster around the Milky Way supermassive black hole. Figure 2 shows the energy conservation and elapsed time for integrations until $t=1$ for the case $N=10^4$ and for various values of η , the accuracy parameter in the Hermite integrator, and r_{crit} , the maximum distance from the black hole at which a particle enters the chain. Post-Newtonian terms were not included (including the PN terms was found to affect the speed of the code only very slightly; they were omitted in order to simplify the discussion of energy conservation). Also shown is the performance of ϕ GRAPE without the regularized chain. The figures show the expected scaling of the Hermite scheme with the accuracy parameter: time steps increase linearly with η making the integration faster and less accurate. For a fourth-order scheme, the relative energy error scales as $\sim dt^5 \sim \eta^{5/2}$, though in the case of the hybrid code the relation is modified by the presence of the chain. Energy conservation generally improves for larger values of r_{crit} , as more and more particles are removed from the N -body integration and are treated more accurately in the chain. The integration time increases rapidly with r_{crit} reflecting the $\sim n_{\text{ch}}^3$ dependence of the chain. Nevertheless it is clear that for all values of η , there exist values of r_{crit} such that the hybrid code is both more accurate and faster, than ϕ GRAPE alone.

Model of the Galactic Center

Our N -body model of the Galactic Center is based on the collisionally relaxed, multi-mass model of Hopman & Alexander (2006) with a steep truncation at $r=0.1$ pc. The model includes the SMBH and four stellar components: main sequence stars, white dwarves, neutron stars, and stellar mass BHs. The total number of stars found in this model is $\sim 75,000$. In addition, we included as test stars five particles with orbital elements corresponding to the five, shortest-period S-stars observed near the galactic center: S0-1, S0-2, S0-16, S0-19, and S0-20.

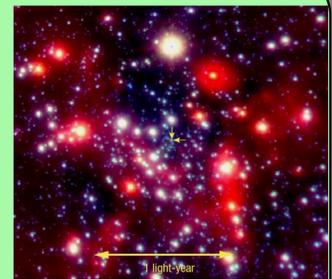


Fig. 3: The Center of the Milky Way (VLT YEPUN + NACO). Credit: ESO

Results

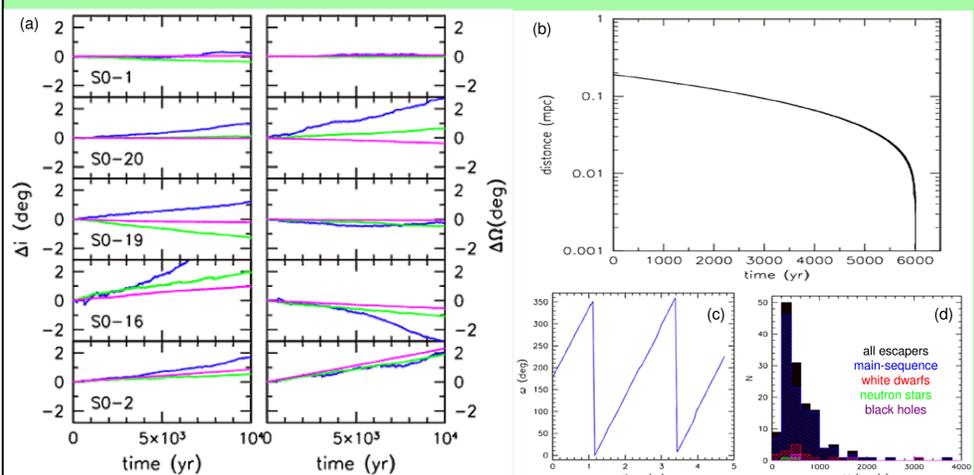


Fig. 4: (a) Evolution of the orbital plane of the S-stars over time in integrations with different particle numbers: $N=10^3$ (magenta), $N=10^4$ (green), $N=7.5 \cdot 10^4$ (blue). (b) Distance evolution with time for an IMBH inspiral. (c) Periastron advancement for the IMBH orbit. (d) Velocity distribution of stars ejected from the center due to the IMBH inspiral.

While evolving our model of the Galactic center we closely follow the orbital evolution of the five S-stars included in the model. On short timescales, the angular momentum of stars like S0-2 should evolve approximately linearly with time due to the (essentially fixed) torques resulting from finite- N departures of the overall potential from spherical symmetry. This evolution is illustrated for all the S-stars in Fig. 4a for three different particle numbers. Plotted are the two Keplerian elements (i, Ω) = (inclination, right ascension of ascending node) that measure the orientation of the orbital planes. These angles would remain precisely constant in any spherical potential and their evolution is due entirely to finite- N departures of the potential from spherical symmetry. Simple arguments can be used to predict that orbital elements should evolve in this regime as a function of the typical perturber mass, N is the number of stars within a sphere of radius a , the semi-major axis of the test star, and the (Keplerian) orbital period $P(a)$. These predictions are quite consistent with our results, e.g. the dependence of the evolution on N is well reproduced.

As a second application, we used ϕ GRAPEch to follow the relativistic inspiral of an intermediate-mass black hole (IMBH) into the Galactic SMBH. Fig. 4b shows the time evolution of the distance between the IMBH and the SMBH in this integration. The timescale for the inspiral is identical to the theoretically predicted one. The stars have no significant effect on the rate of inspiral. Fig. 4c shows the periastron advancement for the IMBH orbit over a time of 4.5 yr, which roughly corresponds to the time for ω to precess 360° twice, again very much in agreement with the theoretical prediction.

Stars that interact strongly with the SMBH-IMBH binary can be ejected from the Galactic center and such ejections are a possible source of the so-called hyper-velocity stars. Fig. 4d shows the distribution of ejection velocities for stars unbound to the SMBH at the end of the IMBH inspiral. The peak of the distribution is at ~ 300 km/s. Interestingly, about 30% of the ejected stars have velocities ≥ 700 km/s, i.e. large enough to escape the bulge and reach the Galactic halo as hyper-velocity stars. About 150 stars are ejected during the inspiral, which results in an average ejection rate of $\sim 20,000$ Myr⁻¹. This rate is somewhat higher than observed in other simulations, presumably because most of the ejections we see are from stars on orbits that intersect the binary at time zero, and many of these stars would have been ejected at earlier times.

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