Methods for Cross-Analyzing Radio and γ–ray Time Series Data

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Fermi Gamma Ray Space Telescope

Special Thanks to Jim Chiang, Jay Norris, Brad Jackson, Roger Blandford, Tony Readhead, Walter Max-Moerbeck, Joey Richards, Vasiliki Pavlidou, Anne Lahteenmaki

Practical Time Series Analysis Methods

- Data Issues
- Light Curve Representations (Data Cells)
- Scatter Plots
- Correlation Functions (Edelson and Krolik)
- Spectra
  - Amplitude (Power)
  - Phase
  - Wavelet Transform (Scalogram)
  - Wavelet Power (Scalegram)
- Structure Functions
- Time-Scale/Time-Frequency Analysis
- Cautions: stationarity, “nonlinearity”, correlations, …

Data Issues

- Transform event data to (x,t) data cells
- Sampling: study interval distribution

  hist(diff(t))
- Get and understand error distribution
  - Random
  - Systematic

Light Curve Representation

◆ Gamma Ray Event data:

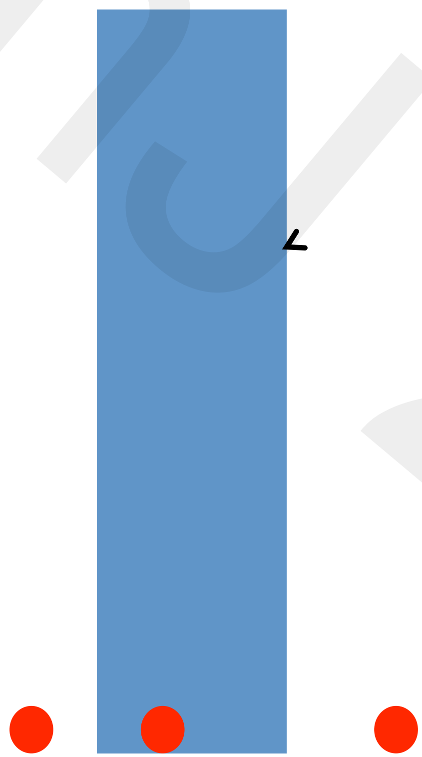
- A cell for each photon ($1/dt_n$, dt_n)
- Independent events?

◆ Flux Measurements:

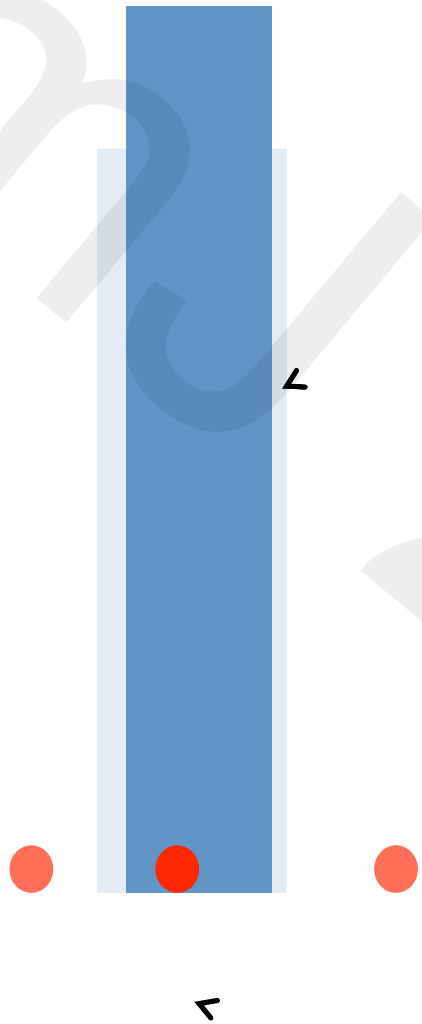
- Data cell trivial: (x_n , t_n)
- Error distribution?

➔Optimal blocks (piecewise constant)

NASA MAX-PLANCK-GESELLSCHAFT RadioNet fermi Gamma-ray Space Telescope



Height = 1 / dt
n / dt
E / dt

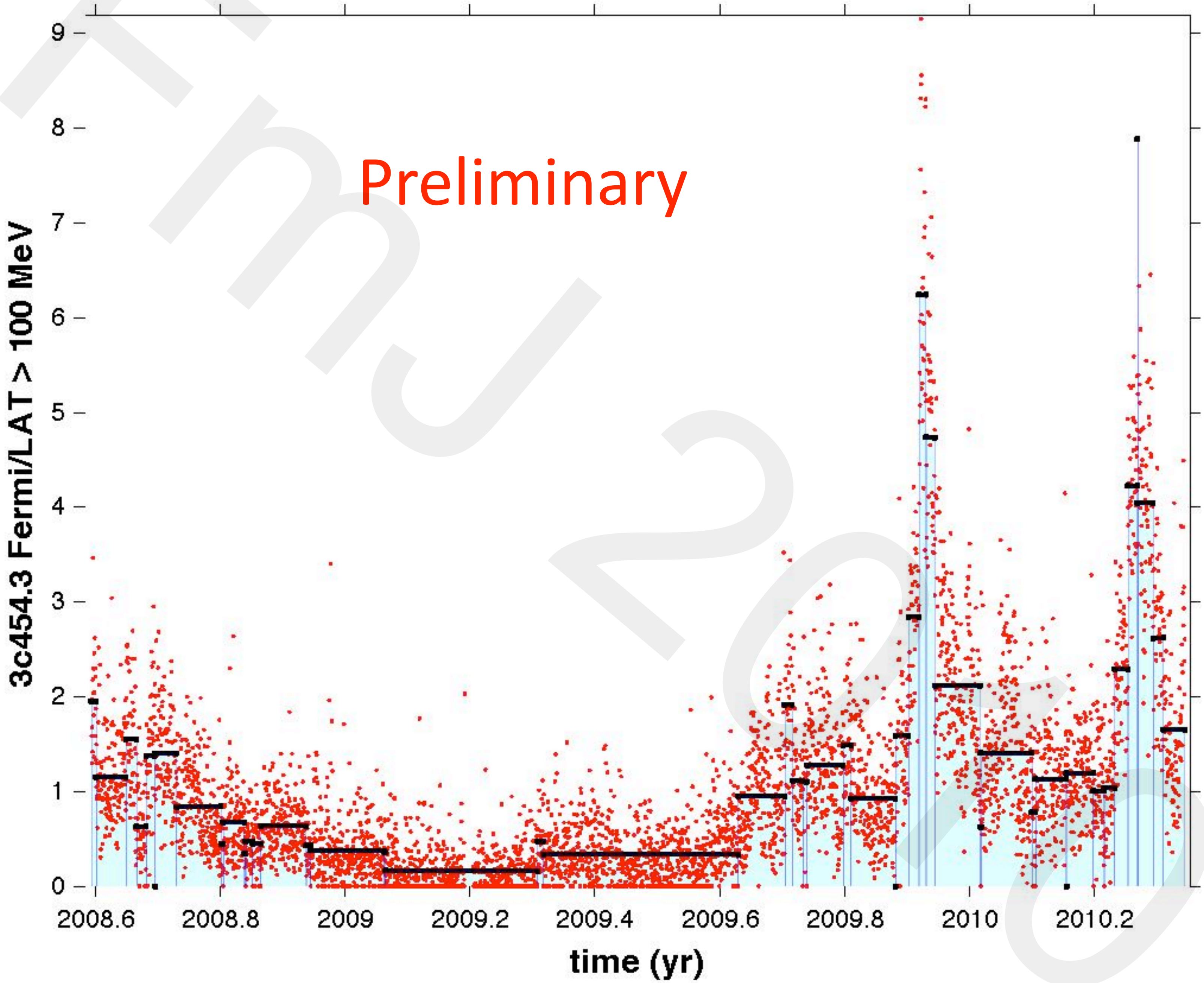


Area = 1 / dt'

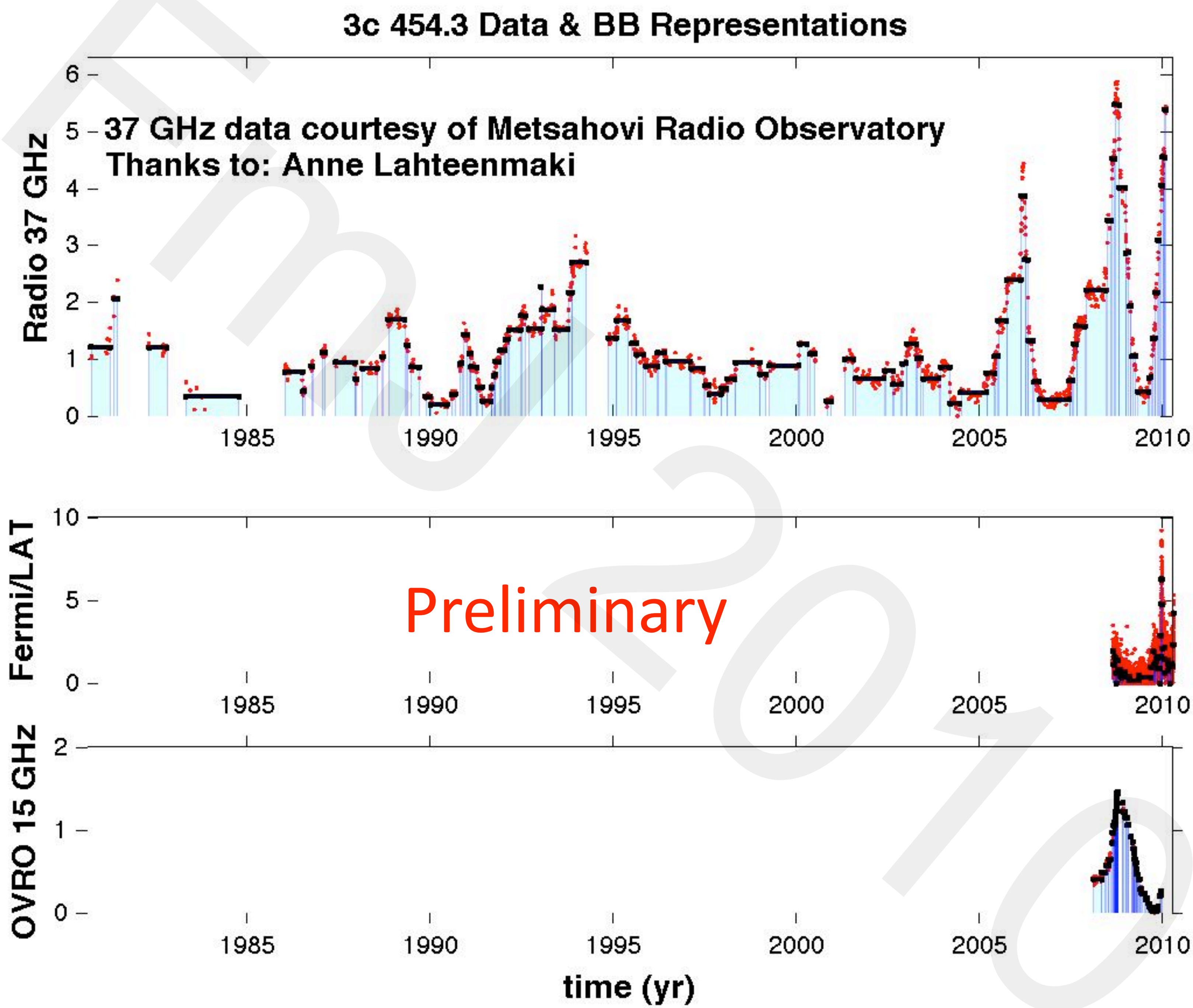
n / dt'

E / dt'

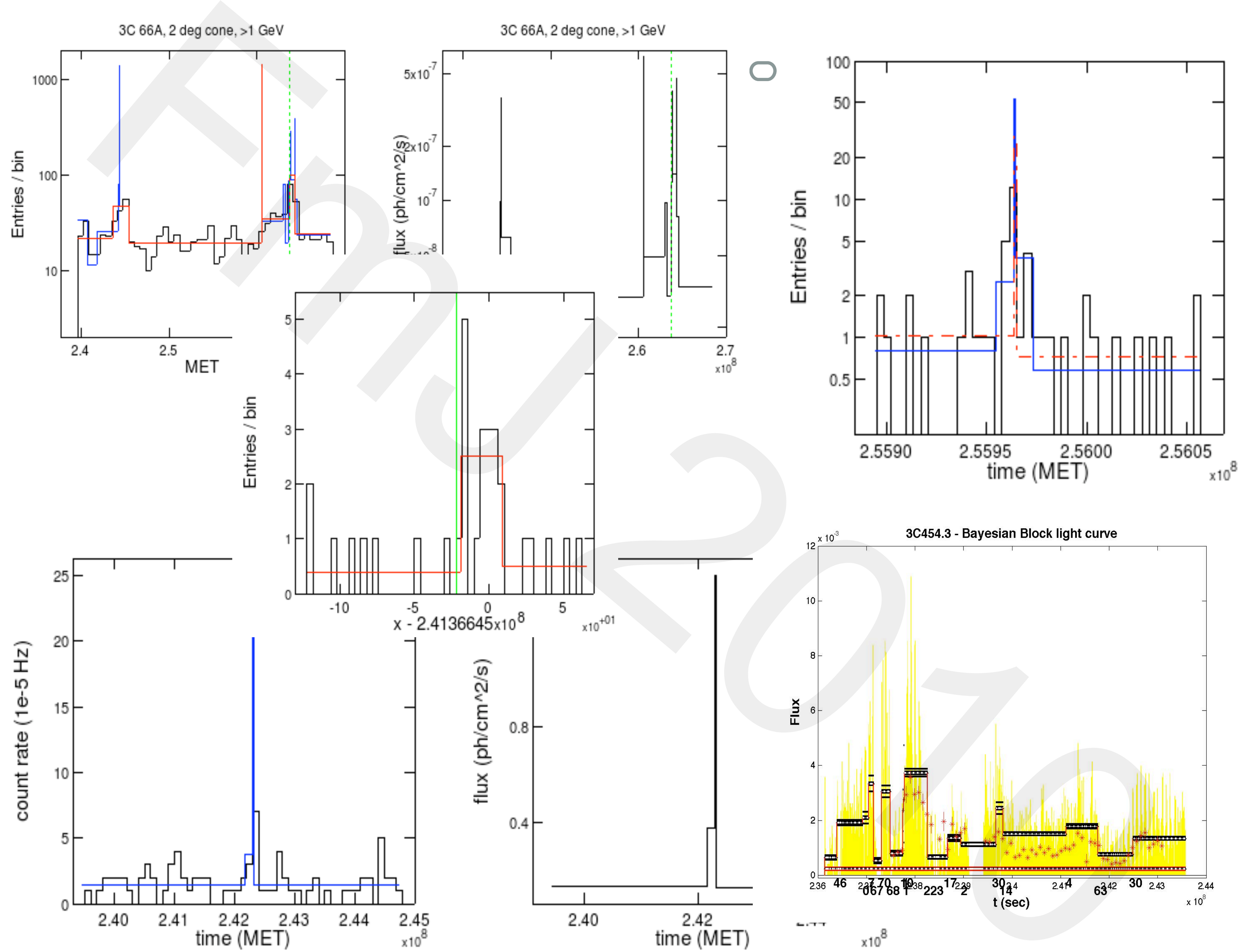
dt' = dt × exposure



Preliminary
3c454.3 Fermi/LAT > 100 MeV
0
1
2
3
4
5
6
7
8
9
2008.6
2008.8
2009
2009.2
2009.4
2009.6
2009.8
2010
2010.2
time (yr)



3c 454.3 Data & BB Representations
37 GHz data courtesy of Metsahovi Radio Observatory
Thanks to: Anne Lahteenmaki
Radio 37 GHz
Fermi/LAT
Preliminary
OVRO 15 GHz
1985
1990
1995
2000
2005
2010
time (yr)



3C 66A, 2 deg cone, >1 GeV
Entries / bin
MET
3C 66A, 2 deg cone, >1 GeV
flux (ph/cm^2/s)
Entries / bin
time (MET)
Entries / bin
x - 2.4136645x10^8
count rate (1e-5 Hz)
time (MET)
flux (ph/cm^2/s)
time (MET)
3C454.3 - Bayesian Block light curve
Flux
t (sec)

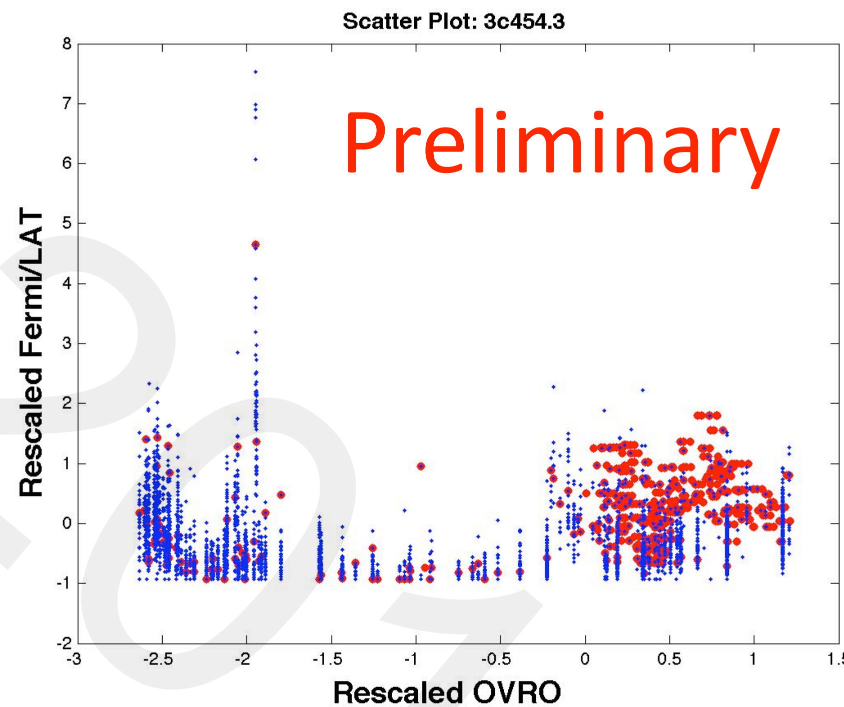
Scatter Plots

Auto-

- Self-dependency
- Introduce lag: X(t+τ) vs X(t)
- Interpolate as needed

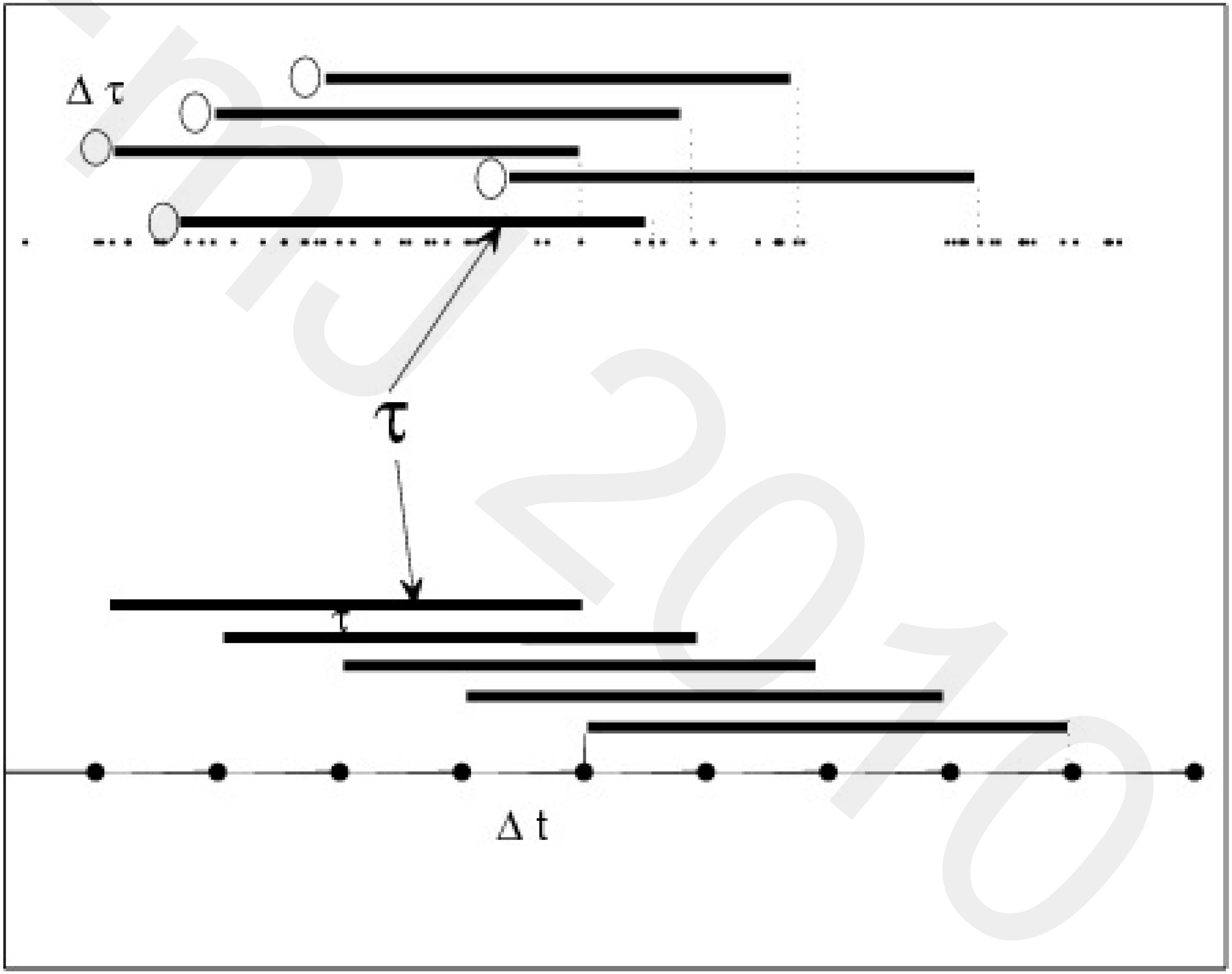
Cross-

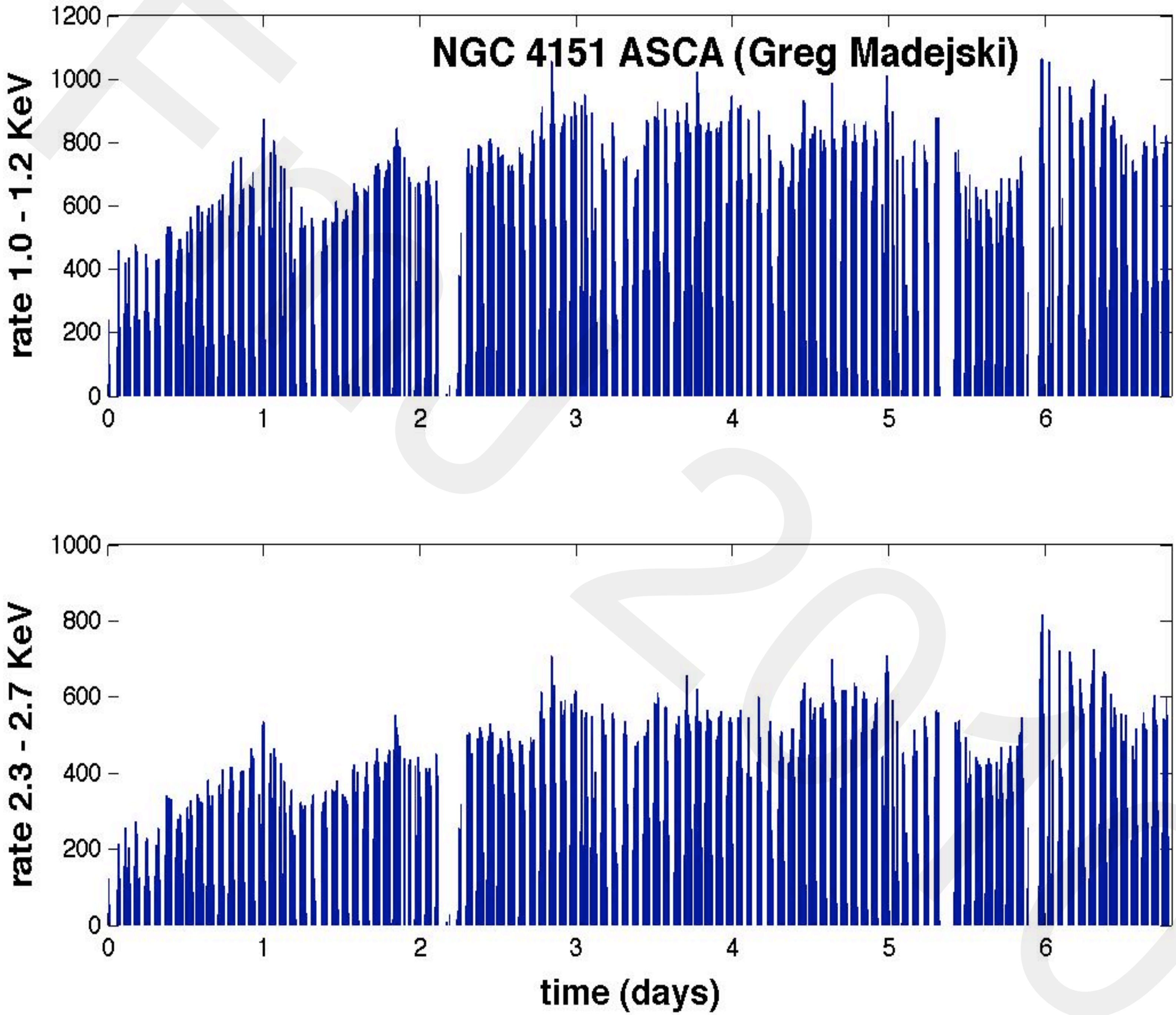
- Joint Probability Distribution
- Introduce lag: X(t+τ) vs Y(t)
- Interpolate X,Y to same times

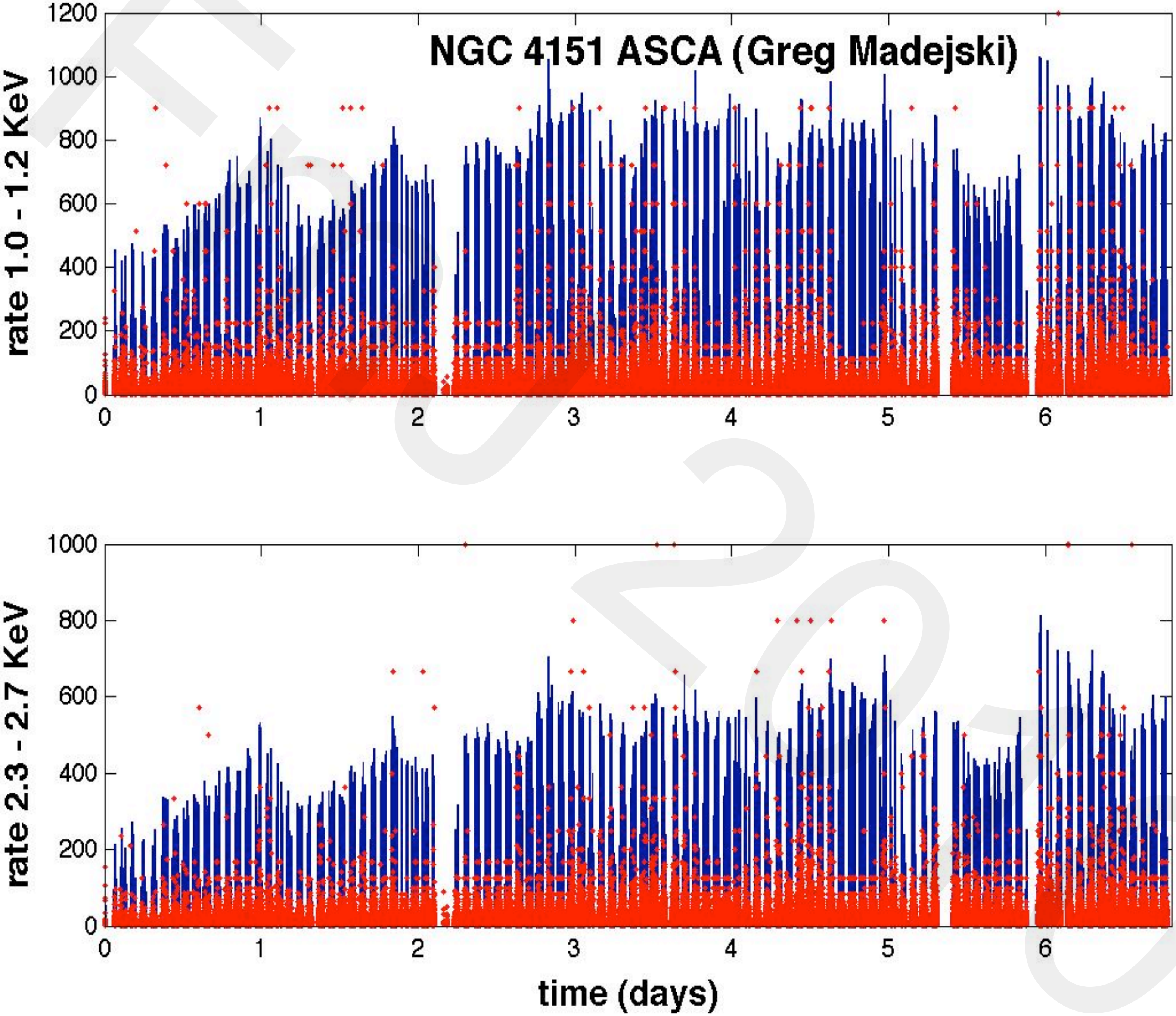


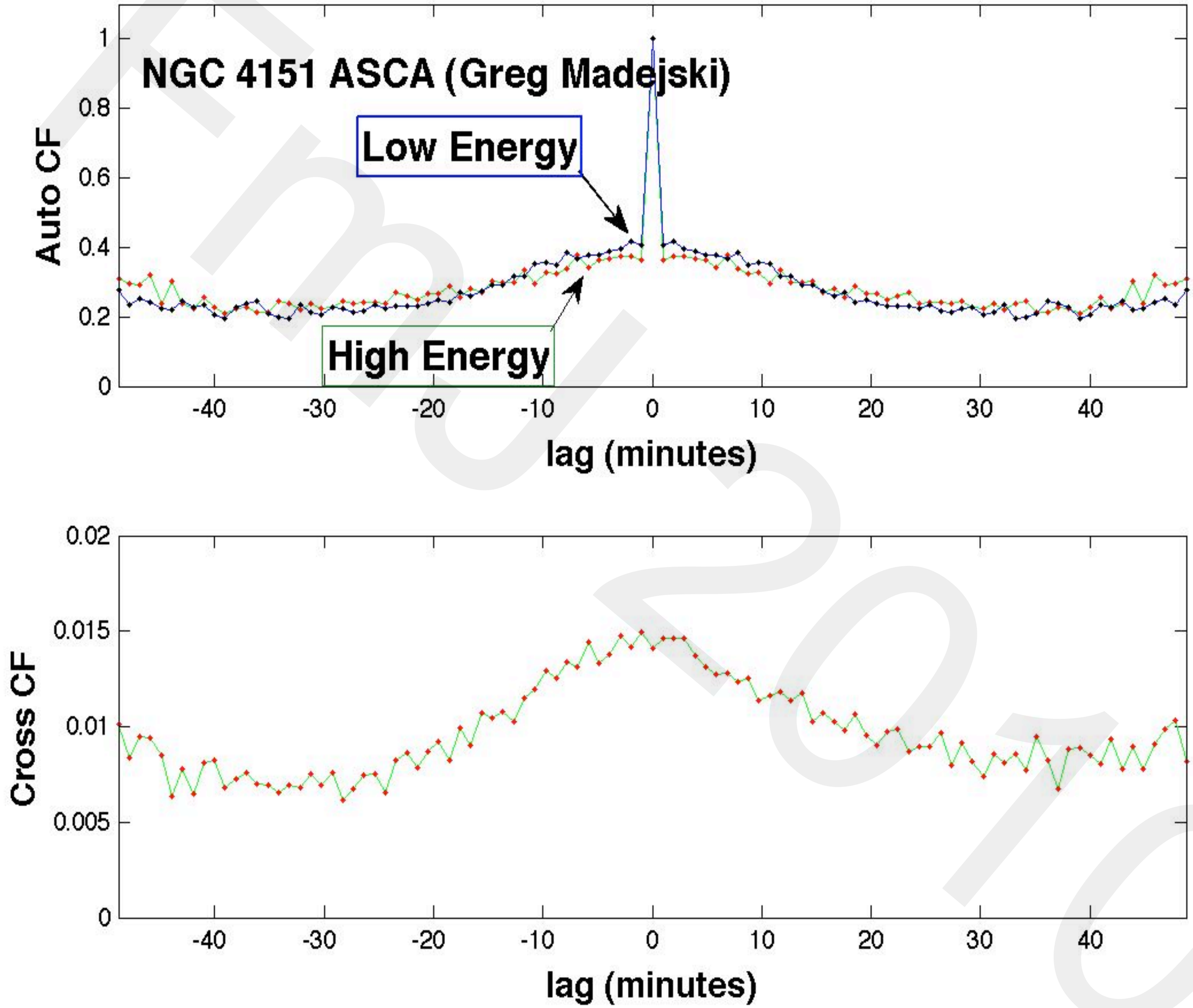
Show Time Sequence!?

Edelson and Krolik: The Discrete Correlation Function: a New Method for Analyzing Unevenly Sampled Variability Data, Ap. J. 333, 1988, 646- starting point for all else!

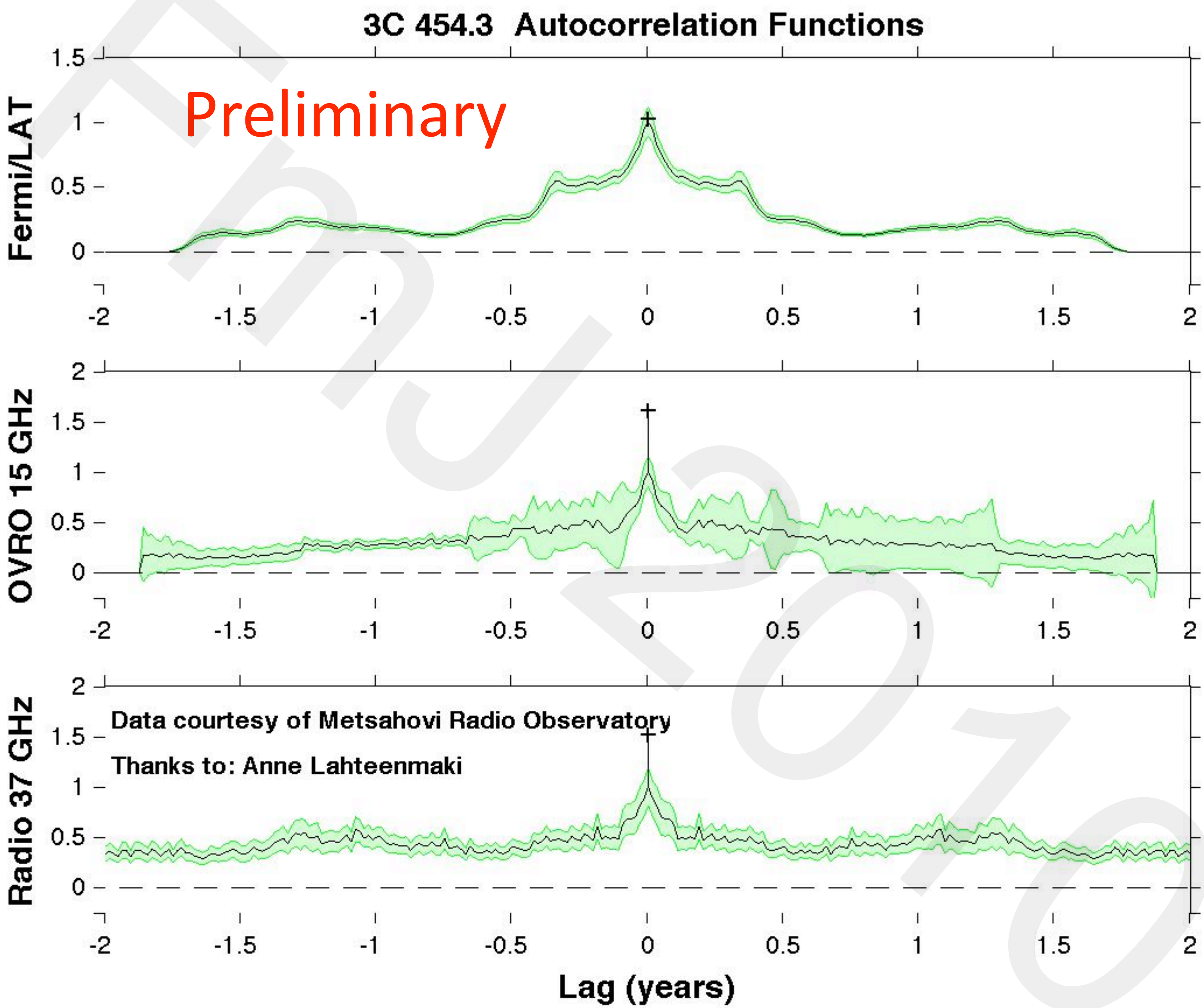



NGC 4151 ASCA (Greg Madejski)
rate 1.0 - 1.2 KeV
1200
1000
800
600
400
200
0
0
1
2
3
4
5
6
rate 2.3 - 2.7 KeV
1000
800
600
400
200
0
time (days)


NGC 4151 ASCA (Greg Madejski)
rate 1.0 - 1.2 KeV
rate 2.3 - 2.7 KeV
time (days)
0
200
400
600
800
1000
1200
1
2
3
4
5
6



NGC 4151 ASCA (Greg Madejski)
Low Energy
High Energy
Auto CF
lag (minutes)
Cross CF
lag (minutes)


3C 454.3 Autocorrelation Functions
Preliminary
Fermi/LAT
OVRO 15 GHz
Radio 37 GHz
Data courtesy of Metsahovi Radio Observatory
Thanks to: Anne Lahteenmaki
Lag (years)
1.5
1
0.5
0
2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
FMJ2010

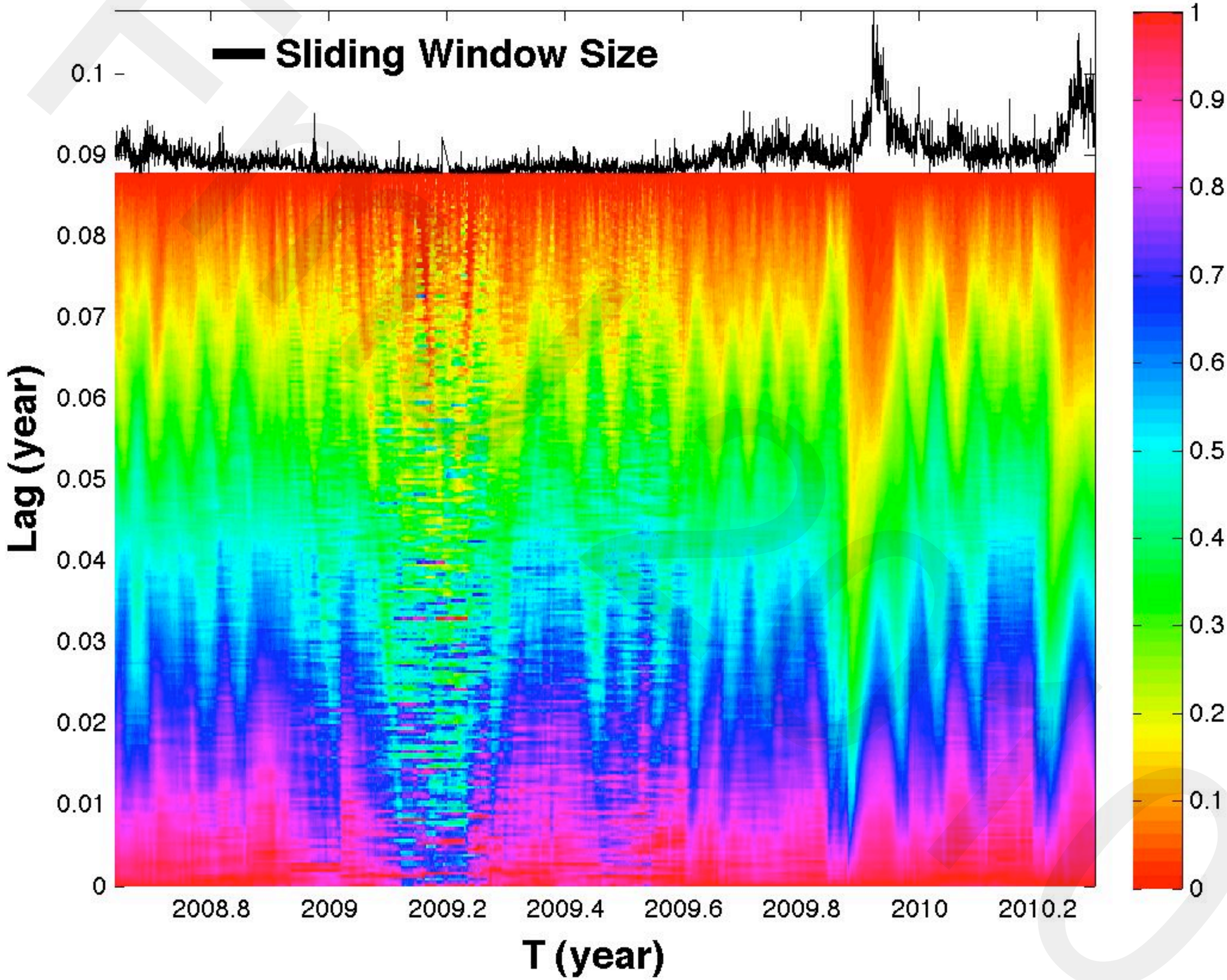
Time-Frequency/Time-Scale Analysis

Transform to a new view of the time series information.

- A Reality in joint time & frequency (or scale) representation
- Atomic decomposition
  - Time-frequency atoms
  - Over-complete representations
  - Optimal Basis Pursuit (Mallat), etc.
- Uncertainty Principle: T-F resolution tradeoff
- Non-stationary processes
  - Flares
  - Trends & Modulations
  - Statistical change-points
- Instantaneous Frequency
- Local vs. Global structure
- Interference (cross-terms in bi-linear representation)

Time-Frequency/Time-Scale Analysis (Temps-Fréquence) Patrick Flandrin http://perso.ens-lyon.fr/patrick.flandrin/publis.html; A Wavelet tour of Signal Processing (Une Exploration des Signaux en Ondelettes) Stéphane Mallat

Preliminary

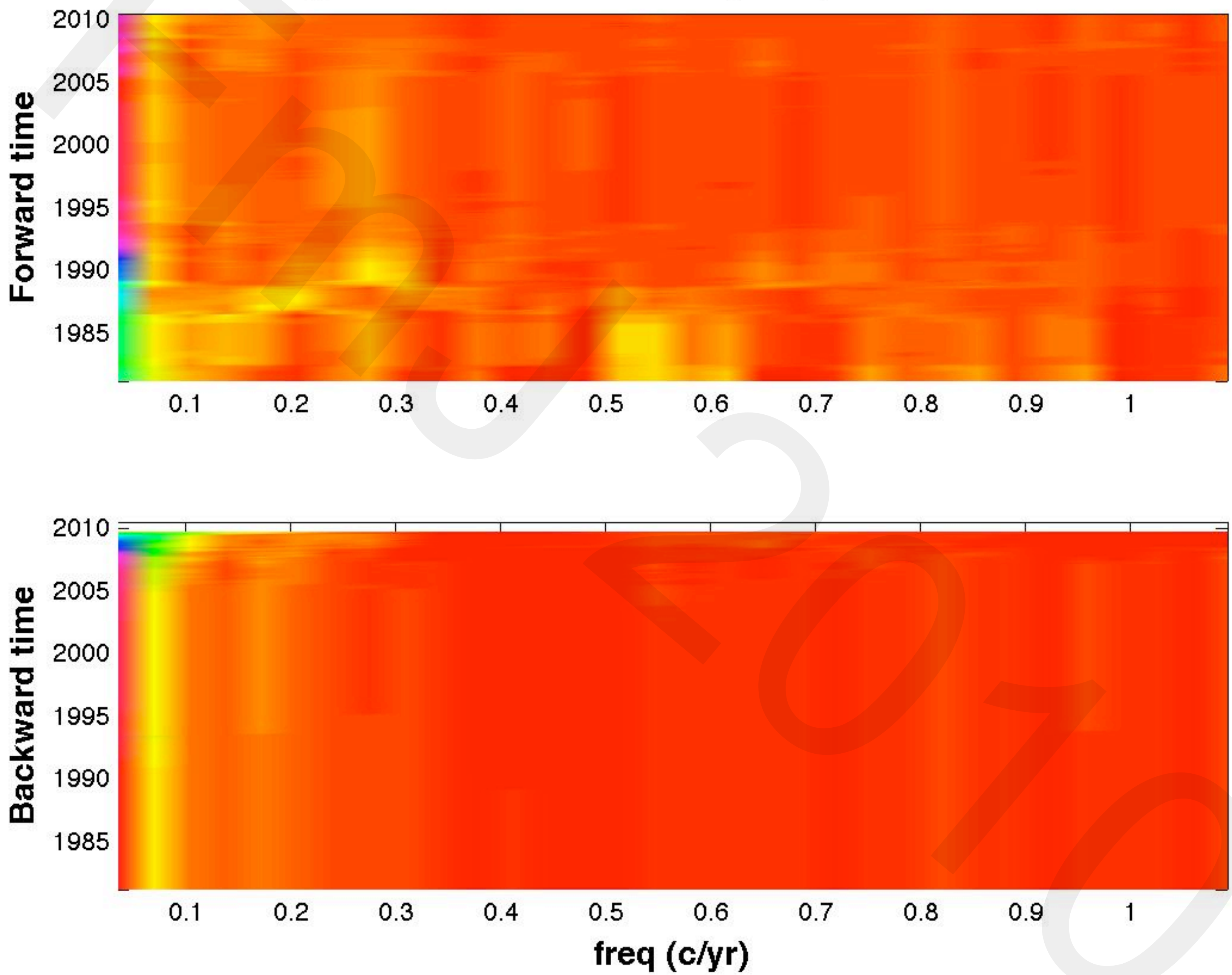


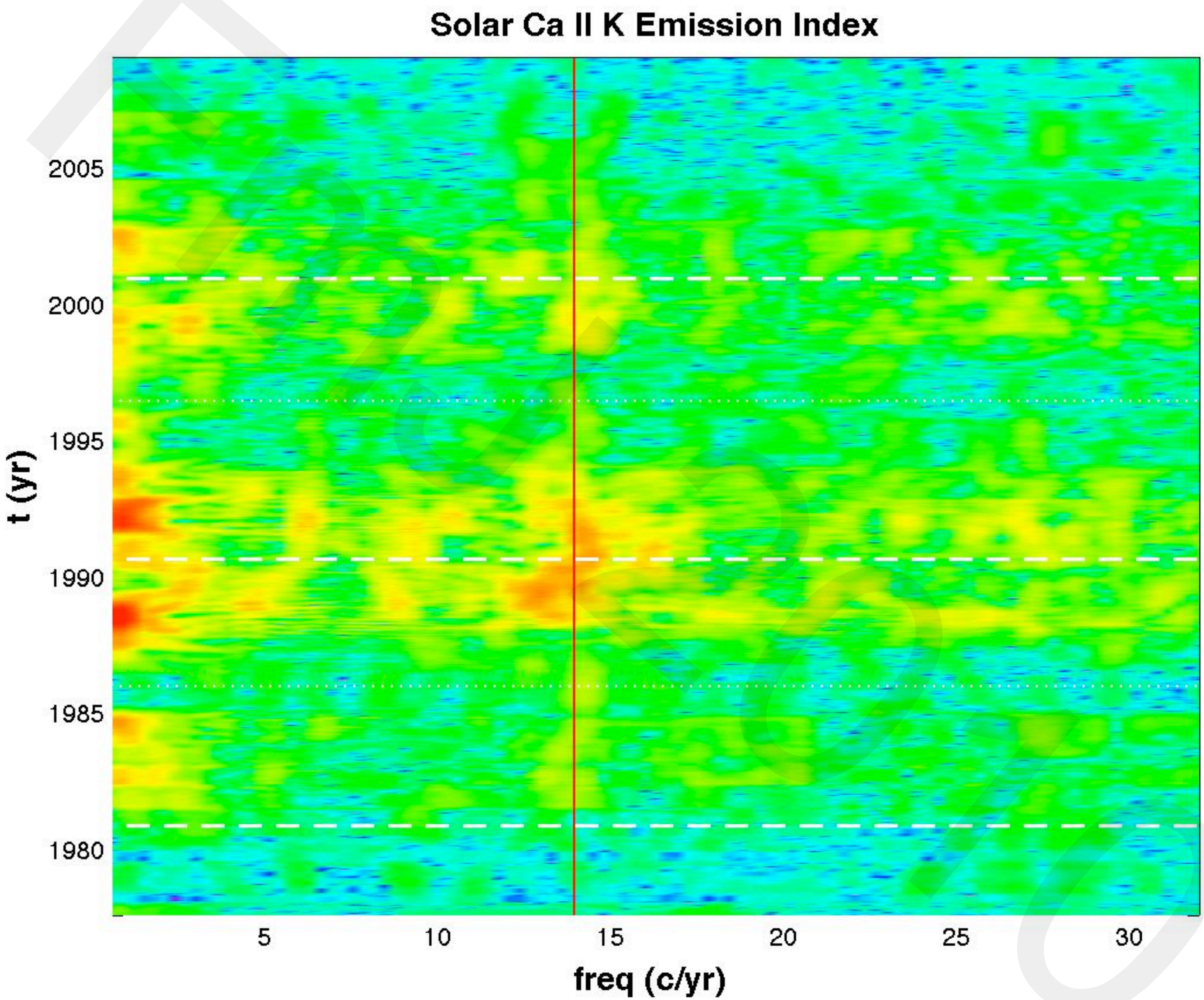
Page-Levin Causal
Time-Frequency Distributions:

$P^{+}(t, \nu) = \partial/\partial t \mid \int_{-\infty}^{t} x(s)\exp(-2\pi i \nu s) \mid^{2}$
(forward)

$P^{-}(t, \nu) = \partial/\partial t \mid \int_{t}^{+\infty} x(s)\exp(-2\pi i \nu s) \mid^{2}$
(backward)

Page-Levin Causal Time-Frequency Distributions

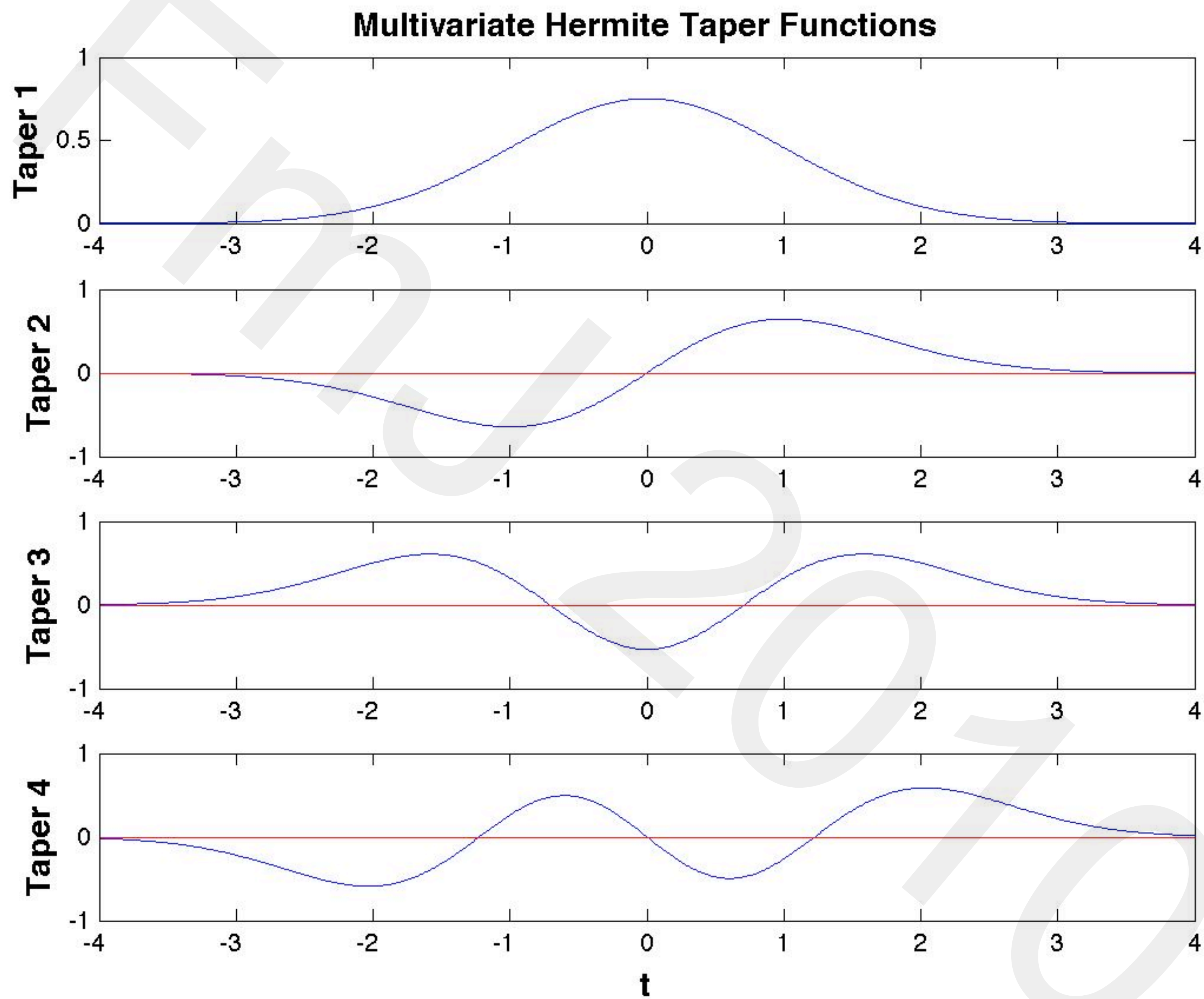



Solar Ca II K Emission Index
t (yr)
2005
2000
1995
1990
1985
1980
5
10
15
20
25
30
freq (c/yr)

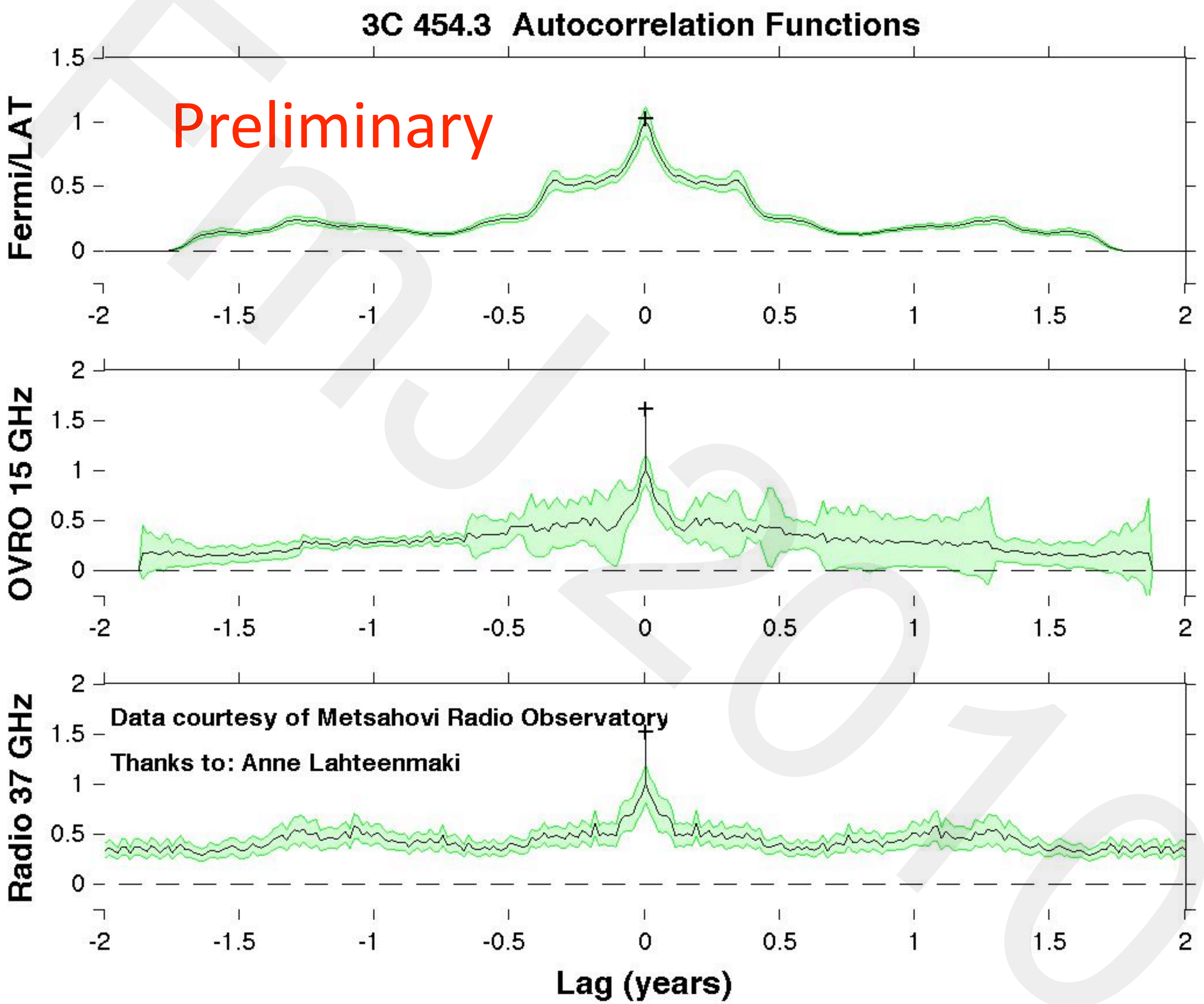
Multi-taper Analysis (Thomson 1982)

- Tapers (windows) reduce sidelobe leakage = bias
- Incomplete use of data → loss of information
- Multitapers recover this information
- Leakage minimization = eigenvalue problem
  - Eigenfunctions: efficient window functions
  - Eigenvalues
    - measure effectiveness
    - determine how many terms to include

Spectral Analysis for Physical Applications: Multitaper and Conventional Univariate Techniques, Don Percival and Andrew Walden (1993)



Multivariate Hermite Taper Functions
Taper 1
Taper 2
Taper 3
Taper 4
1
0.5
0
-1
-4
-3
-2
-1
0
1
2
3
4
t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Preliminary
Fermi/LAT
OVRO 15 GHz
Radio 37 GHz
Data courtesy of Metsahovi Radio Observatory
Thanks to: Anne Lahteenmaki
Lag (years)
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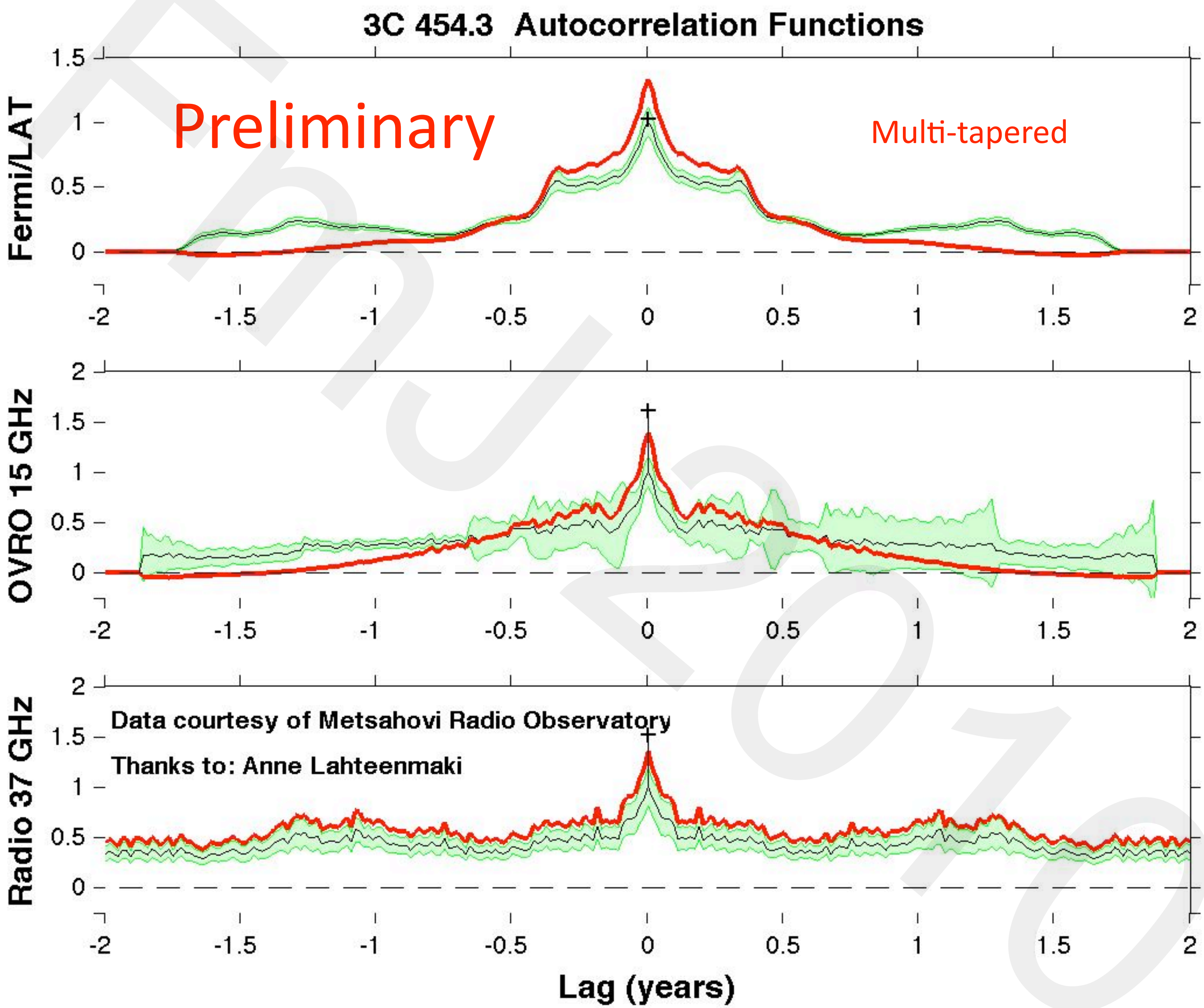
3C 454.3 Autocorrelation Functions
Preliminary
Multi-tapered
Fermi/LAT
OVRO 15 GHz
Radio 37 GHz
Data courtesy of Metsahovi Radio Observatory
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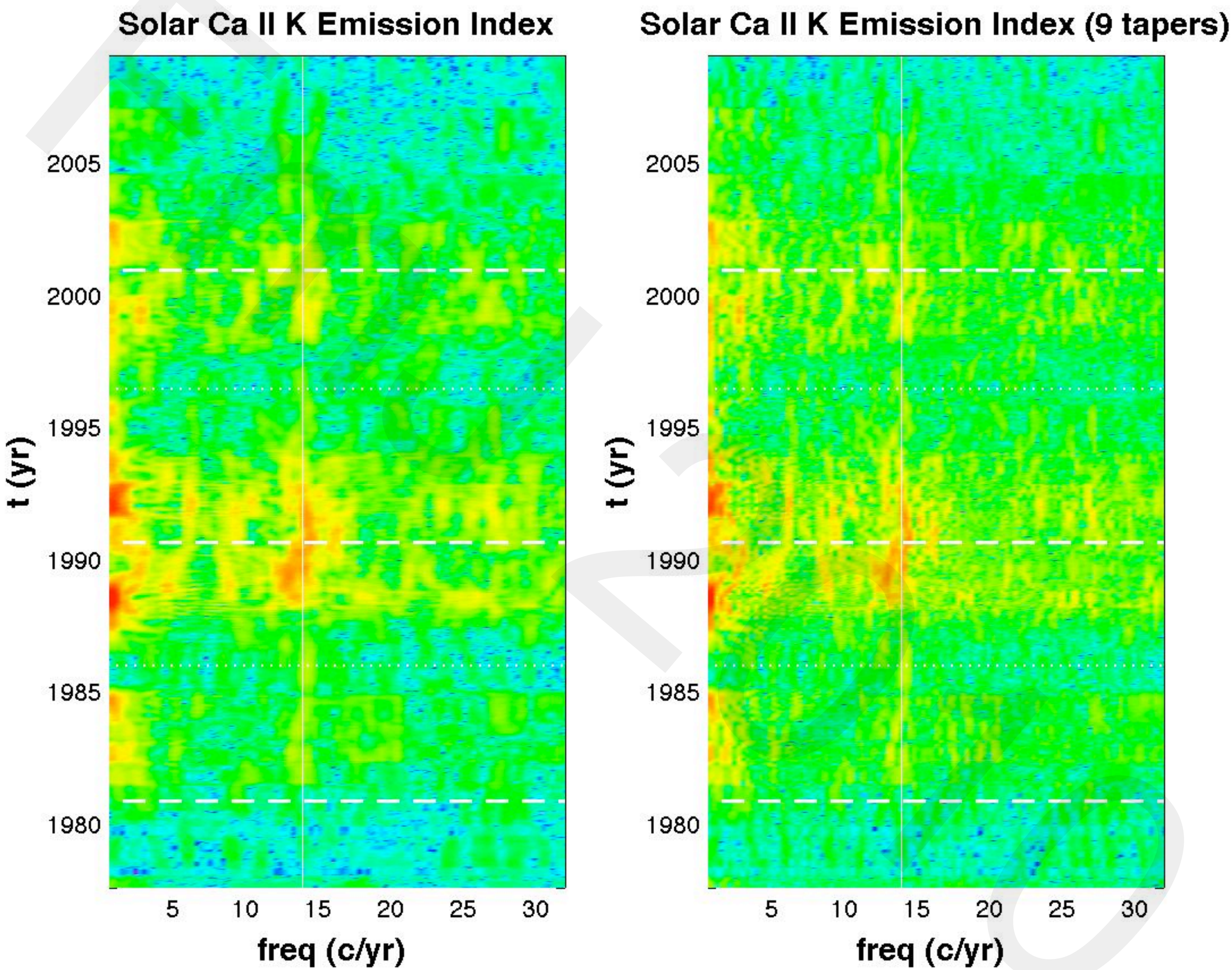
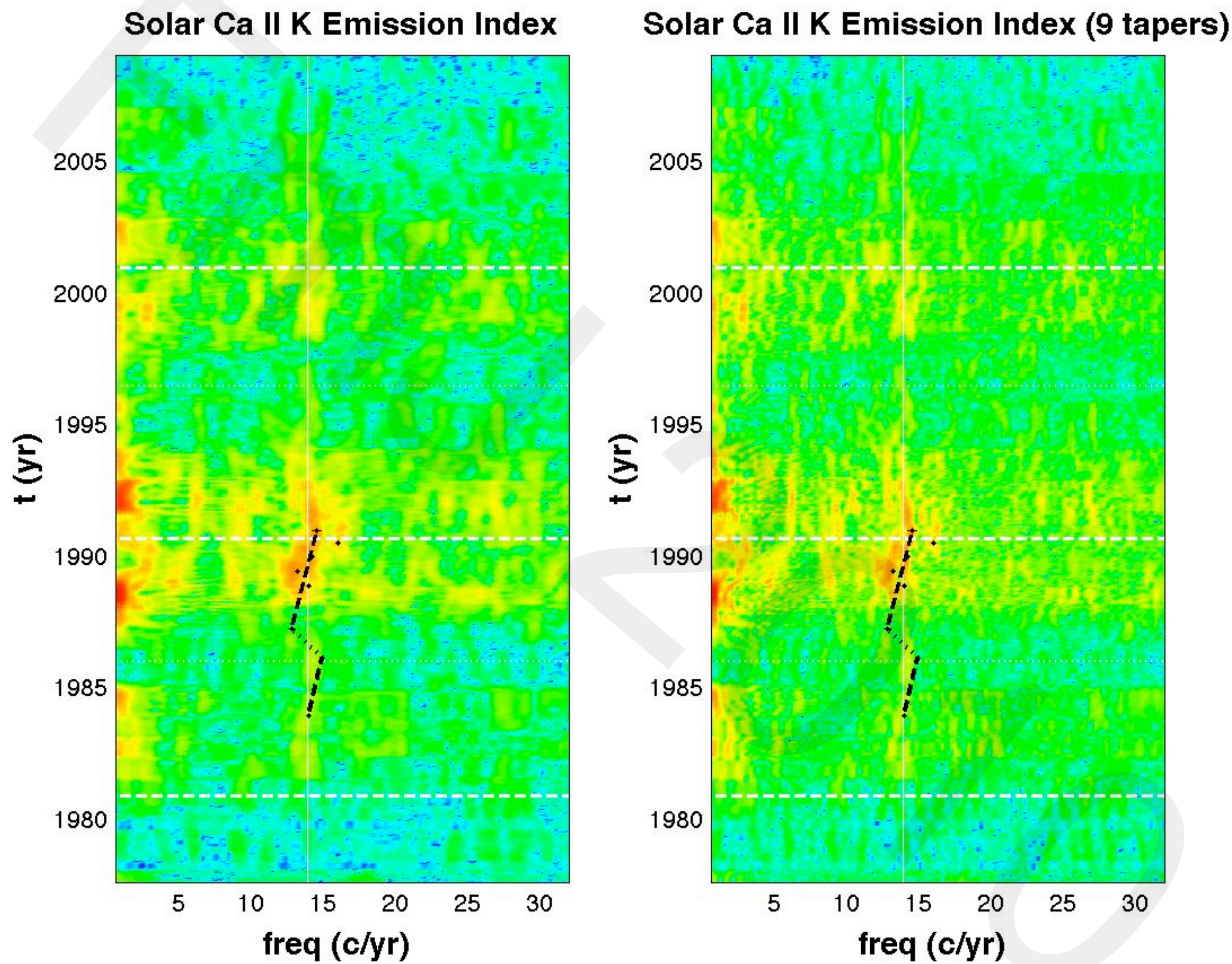
Lag (years)


Solar Ca II K Emission Index
Solar Ca II K Emission Index (9 tapers)
t (yr)
2005
2000
1995
1990
1985
1980
5
10
15
20
25
30
freq (c/yr)


Solar Ca II K Emission Index
Solar Ca II K Emission Index (9 tapers)
t (yr)
2005
2000
1995
1990
1985
1980
freq (c/yr)
5
10
15
20
25
30

Stationarity vs. Non-Stationarity

- Formal definition requires infinite amount of data
- Local stationarity depends on scale
- Construct stationarity measure S[x(t)]
  - E.g. variance of TF distribution vs. time marginal
  - Any such measure has statistical fluctuations
  - Simulate surrogate data: scramble Fourier phase
- Construct distribution of S(surrogate data)

Testing Stationarity with Time-Frequency Surrogates, Jun Xiao, Pierre Borgnat, and Patrick Flandrin

From:
Flandrin & Borgnat
“Revisiting and testing stationarity,” 2008

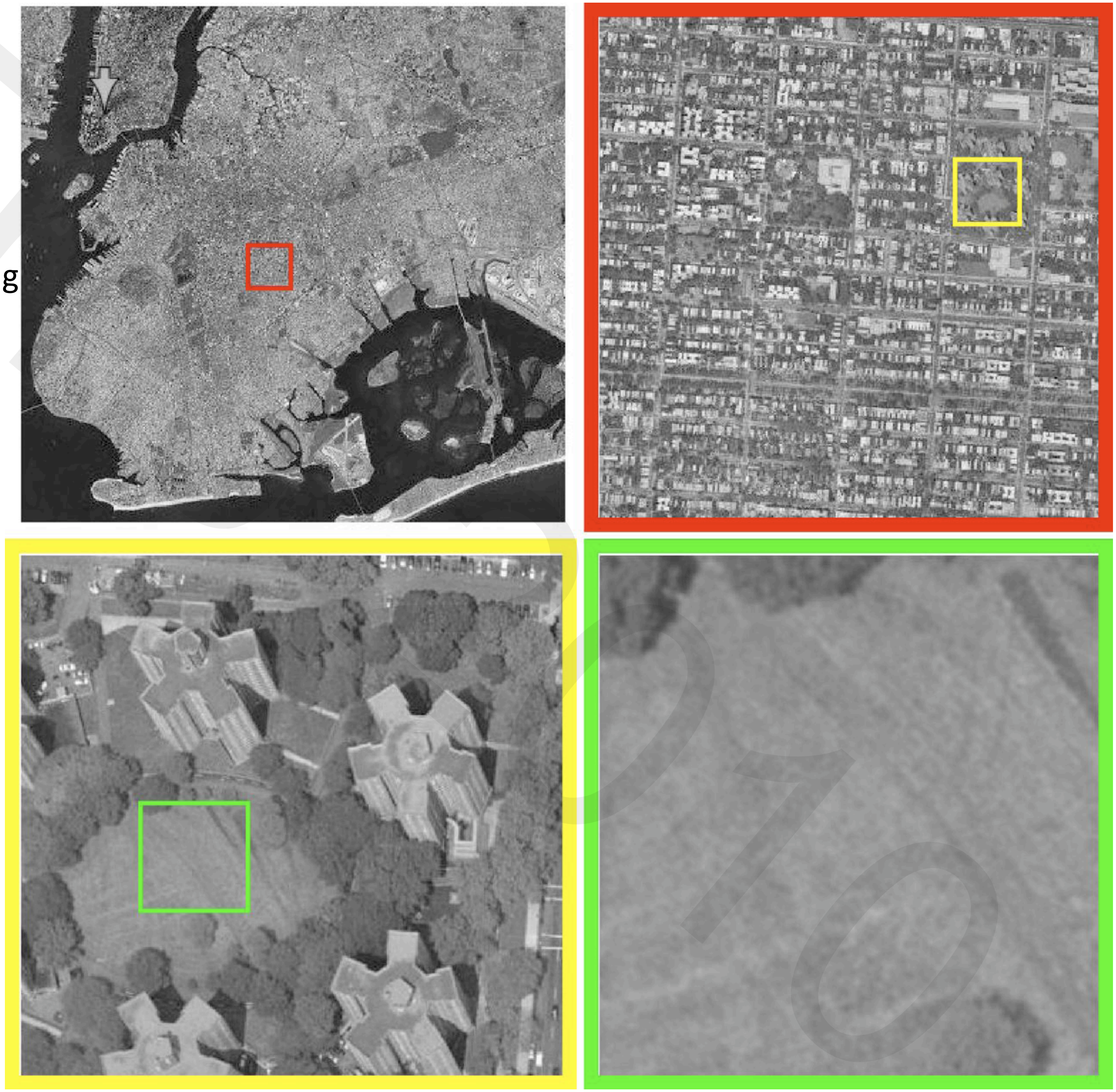
… interpreted as “stationary” or “nonstationary” depending on the observation scale …

TL: nonstationary

TR: stationary (periodic)

BL: nonstationary

BR: stationary (homogeneous texture)



Function	Domain	Range	Auto-	Cross-	Physical Interp
Bayesian blk. Light Curve	Time	Flux	✔	✔ multivar. BB	Flares, events etc.
Scatter Plot	Flux 1	Flux 2		✔	Dependency (not just cor.)
Correlation	Lag	$\langle X^2 \rangle$ $\langle XY \rangle$	✔	✔	Correlated behavior/lags
Spectrum	Frequency	Power	✔	✔	Periodicity 1/f noise …
		Phase	✔	✔	Shifts, lags
Structure	Lag	$\langle X^2 \rangle$ $\langle XY \rangle$	✔	✔	Correlated behavior/lags
Scalogram	Scale/Time	Power	✔	✔	Dynamic behavior
Scalegram	Scale	Power	✔	✔	1/f noise QPOs
Distribution	Time/scale/ frequency	Power	✔	✔	Dynamic behavior

Practical Suggestions

- Study distribution of sample intervals $dt_n = t_{n+1} - t_n$
- Never subtract mean of time series
- Edelson and Krolik CF is the source of all other analysis
- Use self terms in E&K CF to assess observational errors
- Don't confuse source noise with observational noise (doubly stochastic, or Cox processes)
- Source correlation does not imply correlated errors
- H0: All AGN are identical stochastic dynamical systems
- Any stationary random process is exactly shot noise (random pulses; the Wold Decomposition Theorem)
- Linearity is a physical property, not a time series one
- Do not bin data (unless absolutely necessary)

Backup Slides

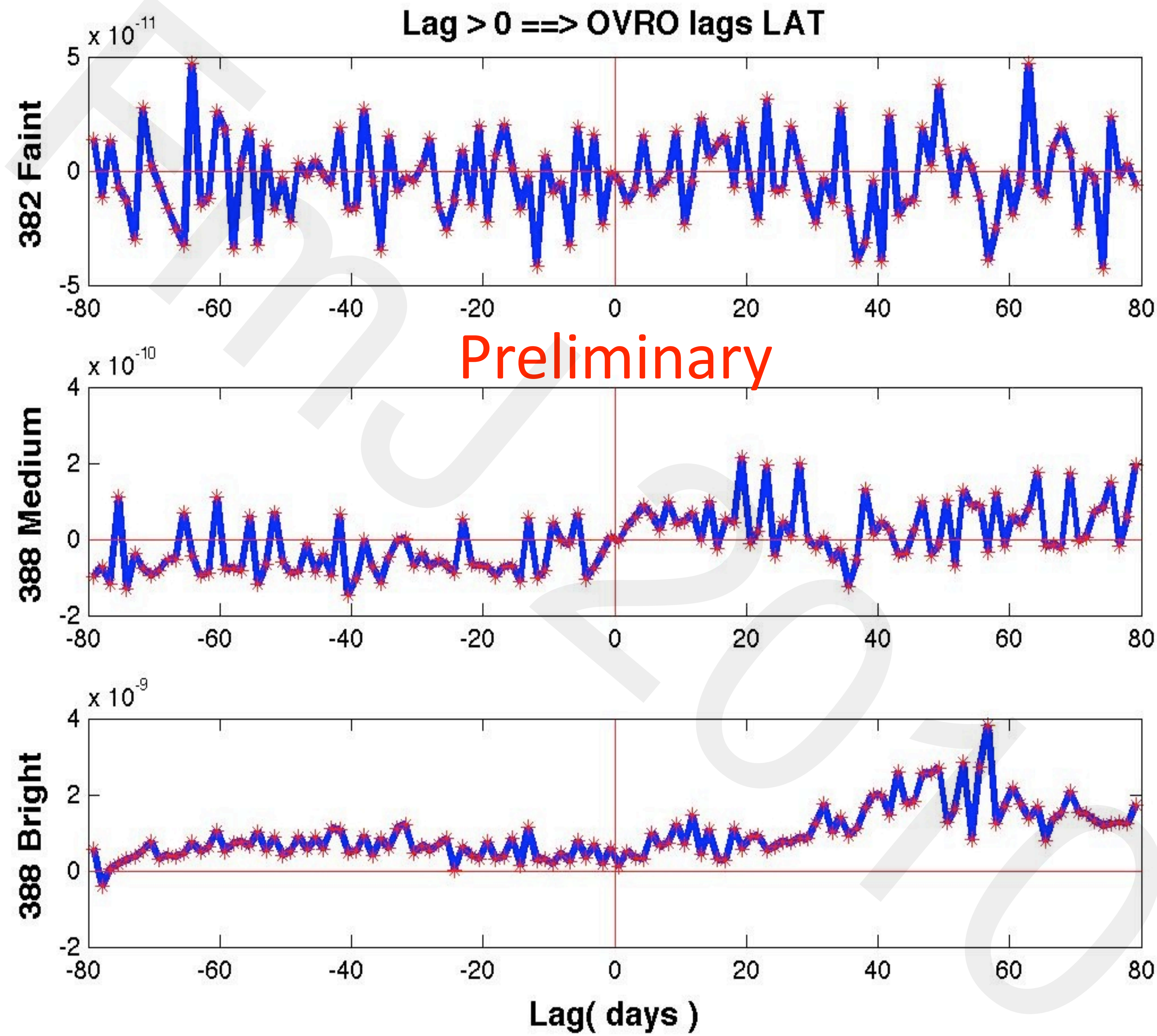
Variable Source → Propagation To Observer → Photon Detection

- Luminosity: random or deterministic
- Photon Emission Independent Random Process (Poisson)

- Random Scintillation, Dispersion, etc.?

- Random Detection of Photons (Poisson)

Correlations in source luminosity <u>do not</u> imply correlations in time series data!


Lag > 0 ==> OVRO lags LAT
x 10^-11
5
0
-5
382 Faint
-80
-60
-40
-20
0
20
40
60
80
Preliminary
x 10^-10
4
2
0
-2
388 Medium
x 10^-9
4
2
0
-2
388 Bright
Lag(days)
FMJ 2010

NASA MAX-PLANCK-GESELLSCHAFT RadioNet Fermi Gamma-ray Space Telescope

All of this will be in the

Handbook of Statistical Analysis of Event Data

… funded by the NASA AISR Program

MatLab Code
Documentation
Examples
Tutorial

Contributions welcome!